Adaptable wire-length distribution with tunable occupation probability

S. Amakawa, T. Uezono, T. Sato,
K. Okada, and K. Masu

Integrated Research Institute
Tokyo Institute of Technology, Japan

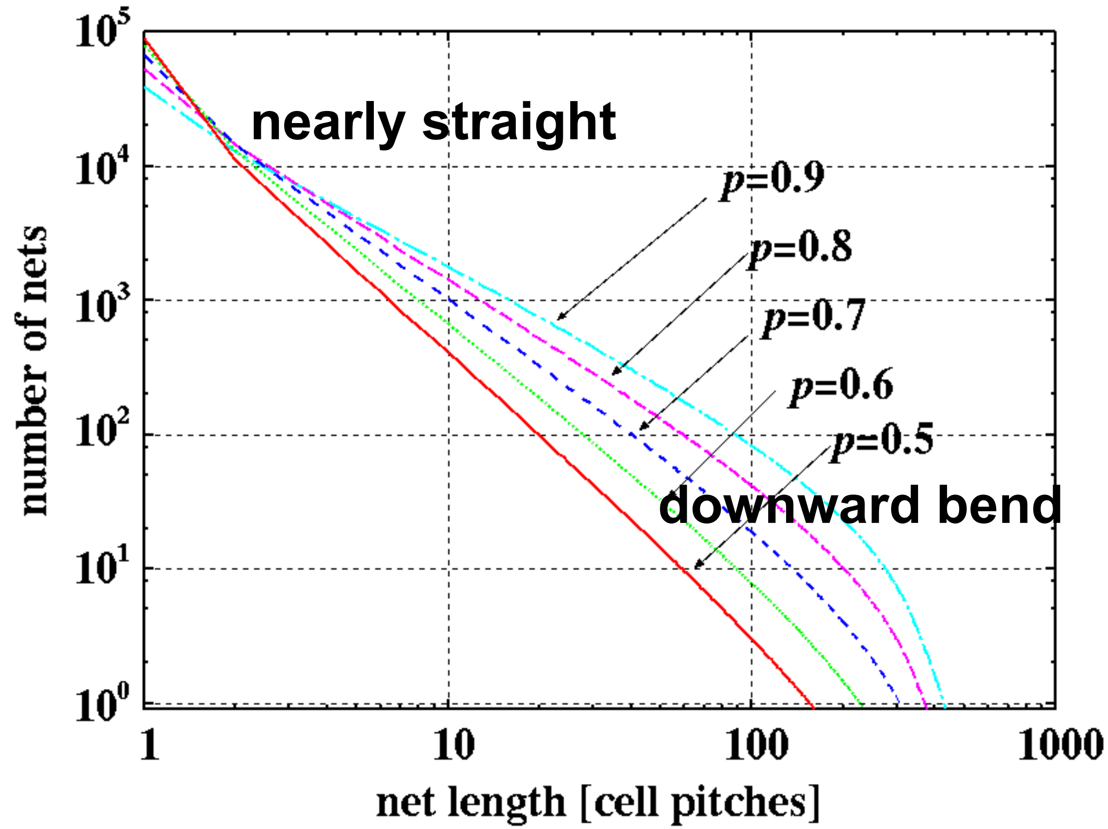
SLIP, March 17, 2007

Outline

- Motivation---Why do we care?
- Flat (non-hierarchical) wire-length distribution models
- Tunable occupation probability
- Comparison with real data
- Conclusions

Motivation

- Typical model wire-length distribution exhibits a nearly straight line (plus a downward bend) on log-log scale
  - Flat model from Ref [1]
  - p is Rent exponent
  - Rent coefficient doesn't affect the shape of the distribution

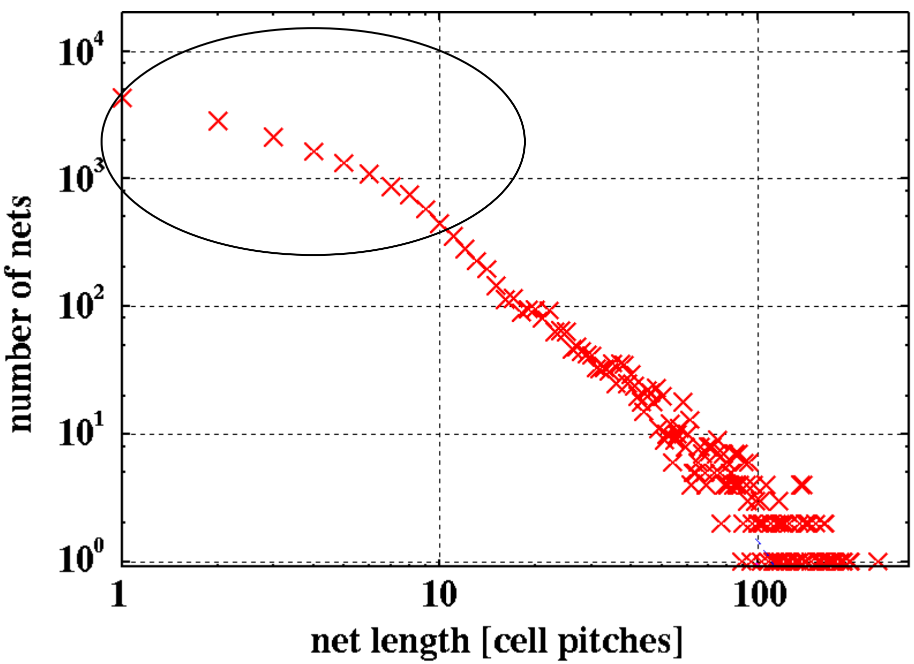


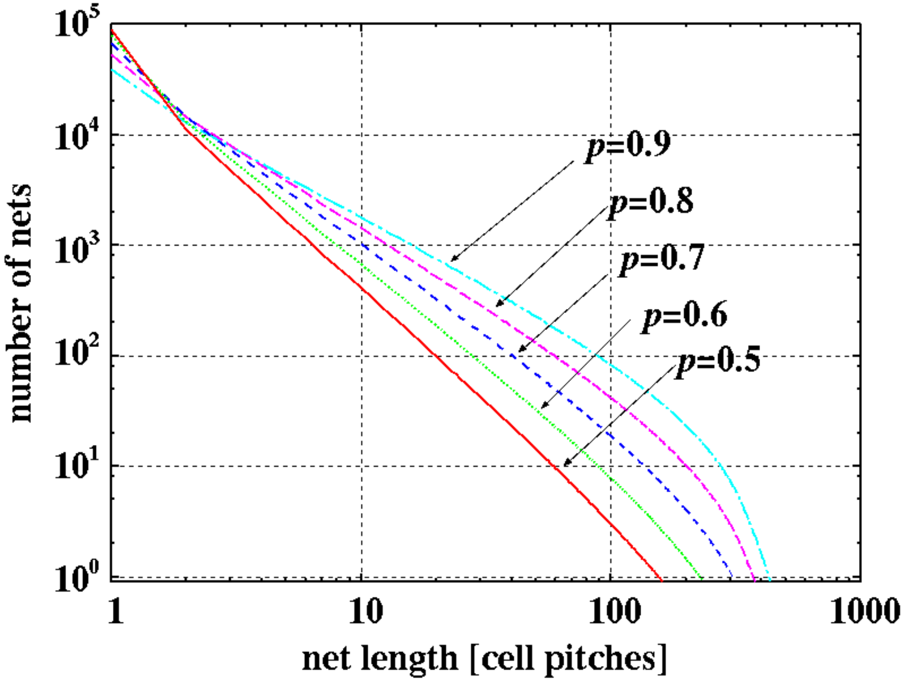
[1] Christie & Stroobandt, "The interpretation and application of Rent's rule," IEEE Trans. VLSI Syst., vol.8, no.6, pp.639-648, 2000.

Motivation

TOKYO TECH Pursuing Excellence

- Real wire-length distribution often doesn't show as many short wires
  - Predicted average wire length could be too short





- Real data like above cannot be expressed by the model shown

Goals

- To develop a wire-length distribution model that can well reproduce real data
  - We want to be sure that we are not far off at the starting point (before starting predicting anything)
- The model must be simple
  - It must be many orders of magnitude faster than placement & routing
  - It must be easy to use

Wire-length distribution function

TOKYO TECH
Pursuing Excellence

- A number of wire-length distribution models are expressed as

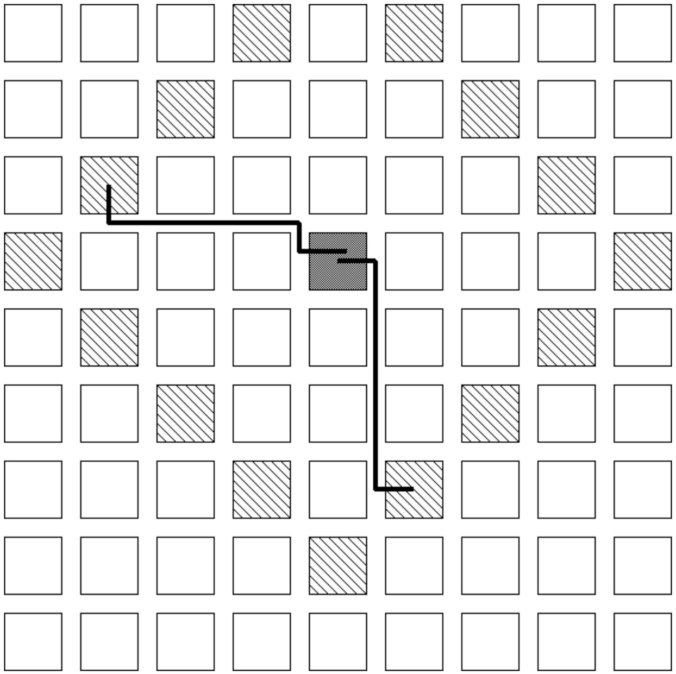
$$w(\ell) = Kq(\ell)D(\ell)$$

$$K = \frac{W_{\text{total}}}{\sum_{\ell=1}^{\ell_{\max}} q(\ell)D(\ell)}$$

W_{total} : Total number of wires

$D(\ell)$: Number of cell pairs

$q(\ell)$: Occupation probability



- Site function $D(\ell)$ depends only on cell arrangement
- Occupation probability $q(\ell)$ is derived with the use of Rent's rule

Solid-state physics analogue

- Wire-length distribution $w(\ell)$: Cell pairs are actually connected by wires according to $q(\ell)$

$$w(\ell) = Kq(\ell)D(\ell)$$

$$K = \frac{W_{\text{total}}}{\sum_{\ell=1}^{\ell_{\max}} q(\ell)D(\ell)}$$

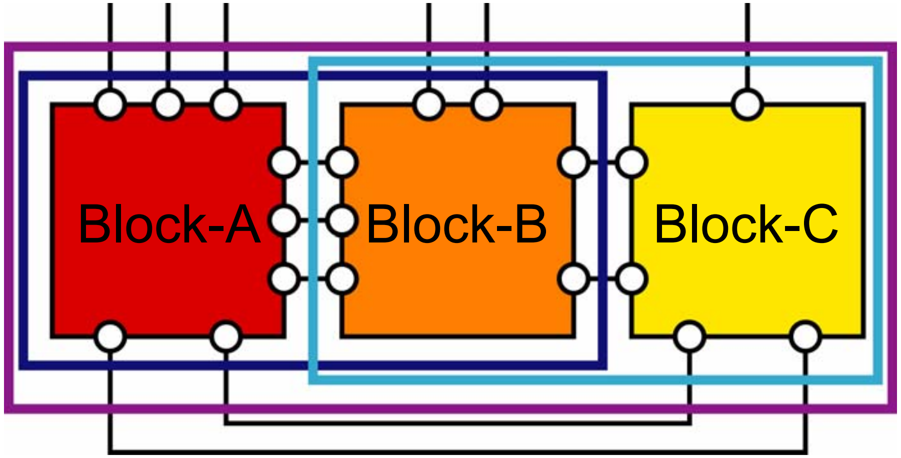
- Electron density $n(\varepsilon)$: Orbitals are filled with electrons according to Fermi distribution $f(\varepsilon)$

$$n(\varepsilon)\mathrm{d}\varepsilon = f(\varepsilon)D(\varepsilon)\mathrm{d}\varepsilon$$

$D(\varepsilon)$: Density of states

$f(\varepsilon)$: Fermi function

Conservation of terminals



$$T_{\mathrm{A}\leftrightarrow\mathrm{C}} = T_{\mathrm{AB}} + T_{\mathrm{BC}} - T_{\mathrm{B}} - T_{\mathrm{ABC}}$$

4 9 8 7 6

- If the number of cells in each block is known

$$T_{\mathrm{A}\leftrightarrow\mathrm{C}} = R(N_{\mathrm{A}} + N_{\mathrm{B}}) + R(N_{\mathrm{B}} + N_{\mathrm{C}}) - R(N_{\mathrm{B}}) - R(N_{\mathrm{A}} + N_{\mathrm{B}} + N_{\mathrm{C}})$$

$R(N) = kN^{p}$ (Rent's rule)

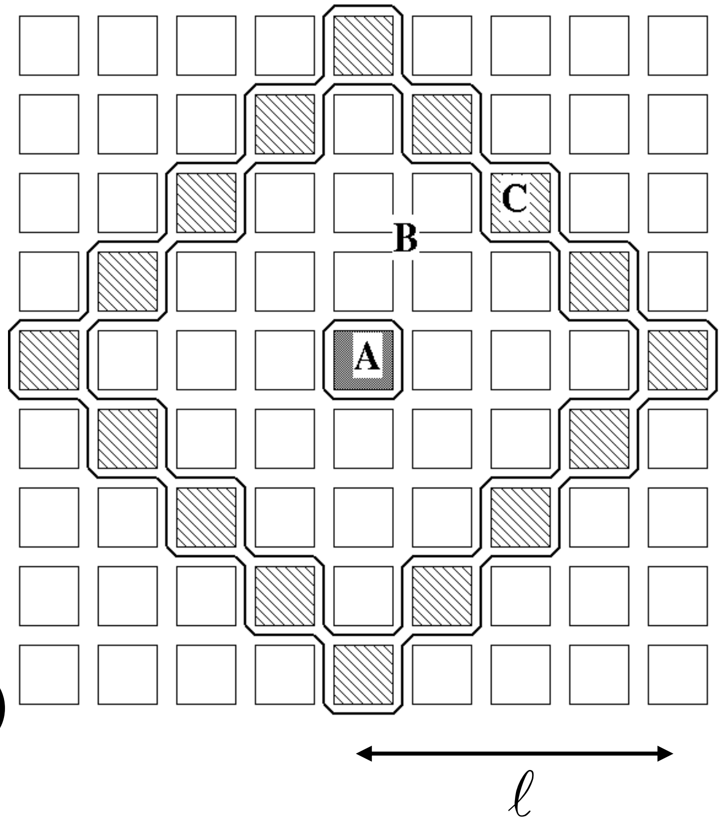
Conservation of terminals

- If blocks A, B, and C are arranged as shown,

$$N_{\mathrm{A}} = 1,\ N_{\mathrm{B}} = 2\ell(\ell - 1),\ N_{\mathrm{C}} = 4\ell$$

$$T_{\mathrm{A}\leftrightarrow\mathrm{B}}(\ell) = R(N_{\mathrm{A}}) + R(N_{\mathrm{B}}) - R(N_{\mathrm{A}} + N_{\mathrm{B}})$$

$$T_{\mathrm{A}\leftrightarrow\mathrm{C}}(\ell) = R(N_{\mathrm{A}} + N_{\mathrm{B}}) + R(N_{\mathrm{B}} + N_{\mathrm{C}}) - R(N_{\mathrm{B}}) - R(N_{\mathrm{A}} + N_{\mathrm{B}} + N_{\mathrm{C}})$$



- Recurrence formula

$$T_{\mathrm{A}\leftrightarrow\mathrm{B}}(\ell + 1) = T_{\mathrm{A}\leftrightarrow\mathrm{B}}(\ell) + T_{\mathrm{A}\leftrightarrow\mathrm{C}}(\ell)$$

Occupation probability function

TOKYO TECH
Pursuing Excellence

- From the recurrence formula and Rent's rule follows

$$\lim_{\ell\to\infty} T_{\mathrm{A}\leftrightarrow\mathrm{B}}(\ell) = \lim_{\ell\to\infty} \sum_{r=1}^{\ell-1} T_{\mathrm{A}\leftrightarrow\mathrm{C}}(r) = k$$

- A probability function can be introduced by

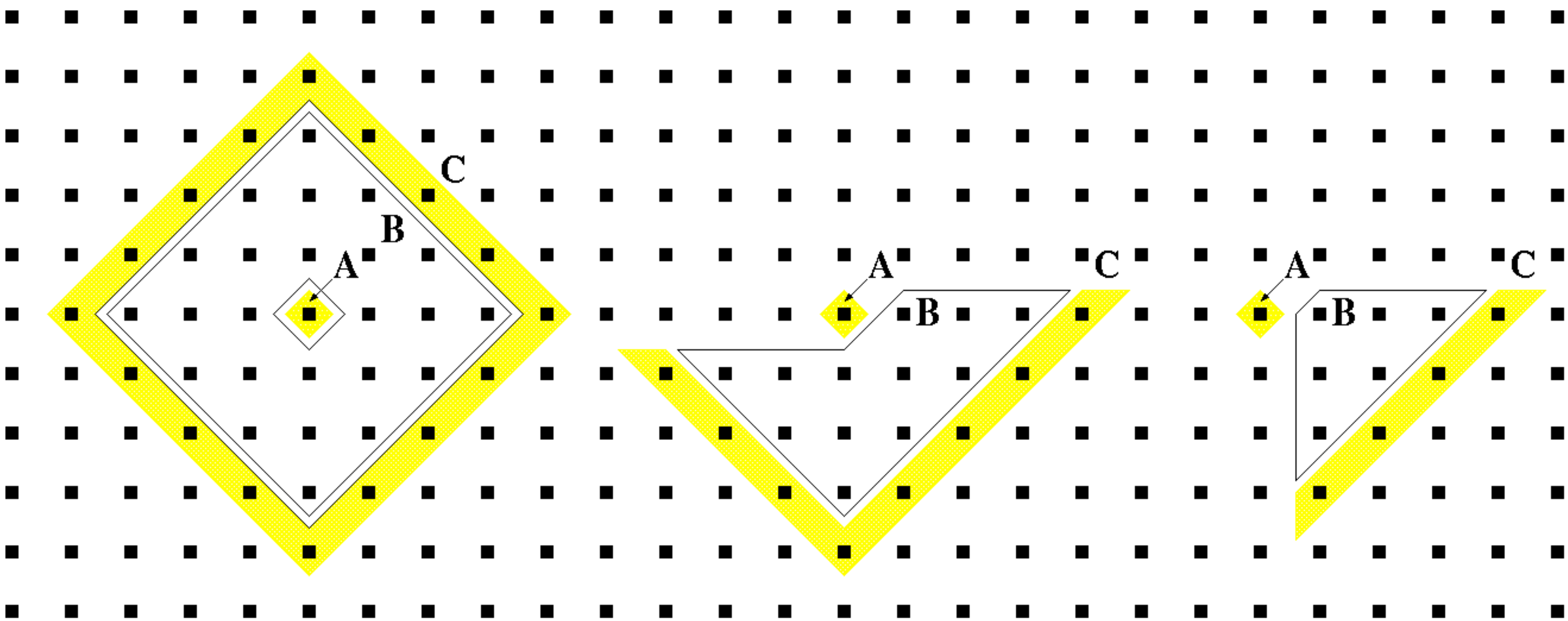
$$\mathrm{prob}_{\mathrm{A}\leftrightarrow\mathrm{C}}(\ell) = \frac{1}{k} T_{\mathrm{A}\leftrightarrow\mathrm{C}}(\ell)$$

- Occupation probability function $q(\ell)$ is defined by

$$q(\ell) = \frac{1}{2\ell}\,\mathrm{prob}_{\mathrm{A}\leftrightarrow\mathrm{C}}(\ell)$$

Configuration for defining $q(\ell)$

- All the following are valid configurations in defining $q(\ell)$



full Manhattan circle	half Manhattan circle	quarter Manhattan circle
$N_{\mathrm{A}}=1$	$N_{\mathrm{A}}=1$	$N_{\mathrm{A}}=1$
$N_{\mathrm{B}}=2\ell(\ell-1)$	$N_{\mathrm{B}}=\ell(\ell-1)$	$N_{\mathrm{B}}=\ell(\ell-1)/2$
$N_{\mathrm{C}}=4\ell$	$N_{\mathrm{C}}=2\ell$	$N_{\mathrm{C}}=\ell$

Configuration for defining $q(\ell)$

- Ref [1] uses

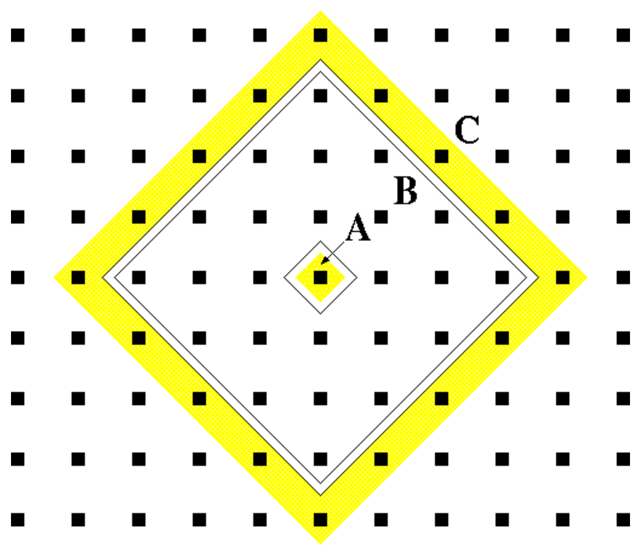
  $$N_{\mathrm{A}} = 1,\ N_{\mathrm{B}} = 2\ell(\ell - 1),\ N_{\mathrm{C}} = 4\ell$$

  corresponding to full Manhattan circle configuration

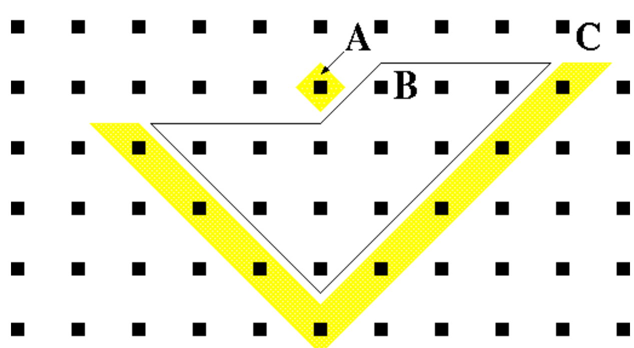
- Ref [2] uses

  $$N_{\mathrm{A}} = 1,\ N_{\mathrm{B}} = \ell(\ell - 1),\ N_{\mathrm{C}} = 2\ell$$

  corresponding to half Manhattan circle configuration



full Manhattan circle



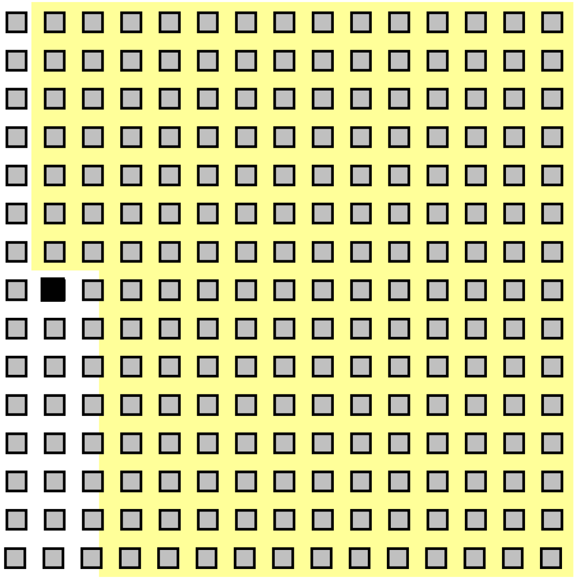
half Manhattan circle

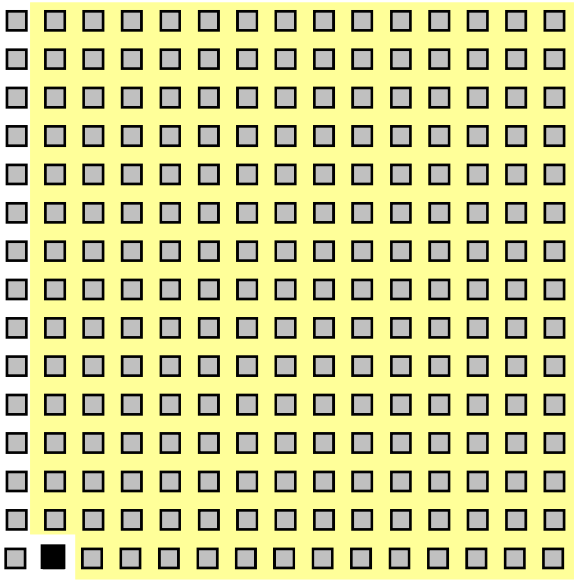
[1] Christie & Stroobandt, IEEE Trans. VLSI Syst., vol.8, no.6, pp.639-648, 2000.

[2] Davis, De, Meindl, IEEE Trans. Electron Devices, vol.45, no.3, pp.580-589, 1998.

Which configuration to use?

- If a cell under consideration is near an edge, half Manhattan should be appropriate
- Likewise, if a cell is near a corner, quarter Manhattan circle should be used
- What is the best configuration on average?





Introducing parameter ζ

- Cell numbers for a generalized partial Manhattan circle configuration

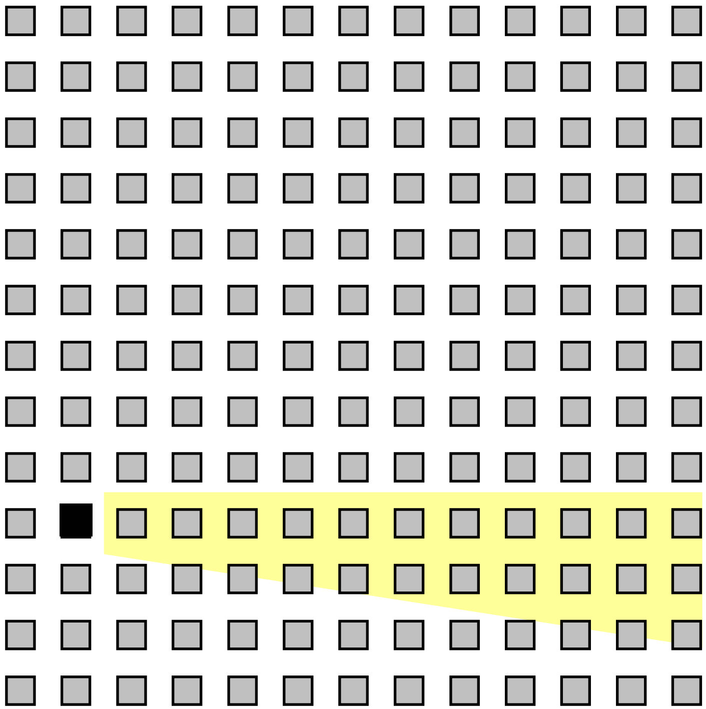
$$N_{\mathrm{A}} = 1,\ N_{\mathrm{B}} = 2\zeta\ell(\ell - 1),\ N_{\mathrm{C}} = 4\zeta\ell$$

$$(0 < \zeta \leq 1)$$

$\zeta = 1$: Full Manhattan circle

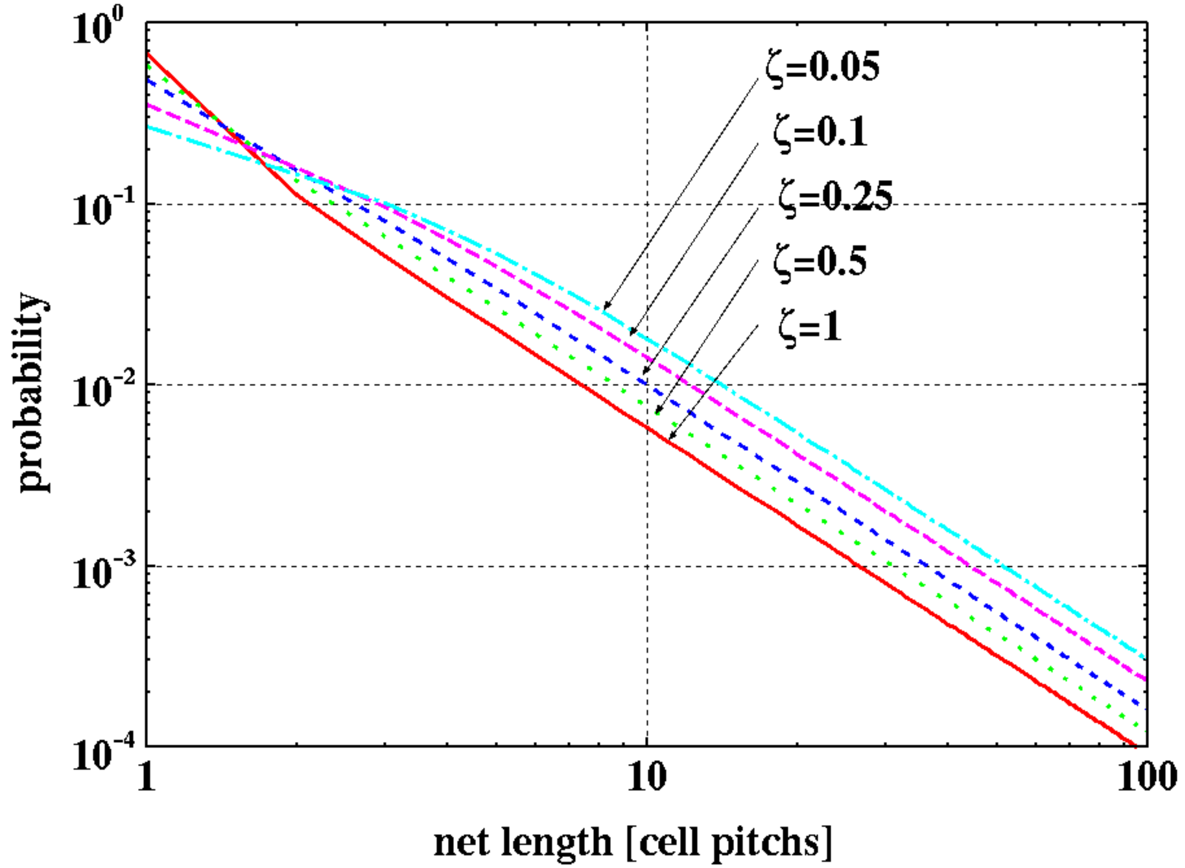
$\zeta = 0.5$: Half Manhattan circle

$\zeta = 0.25$: Quarter Manhattan circle



Introducing parameter ζ

- $\mathrm{prob}_{A\leftrightarrow C}(\ell)$ shows smaller ζ has the desired effect of reducing the numbers of short wires



Another interpretation of ζ

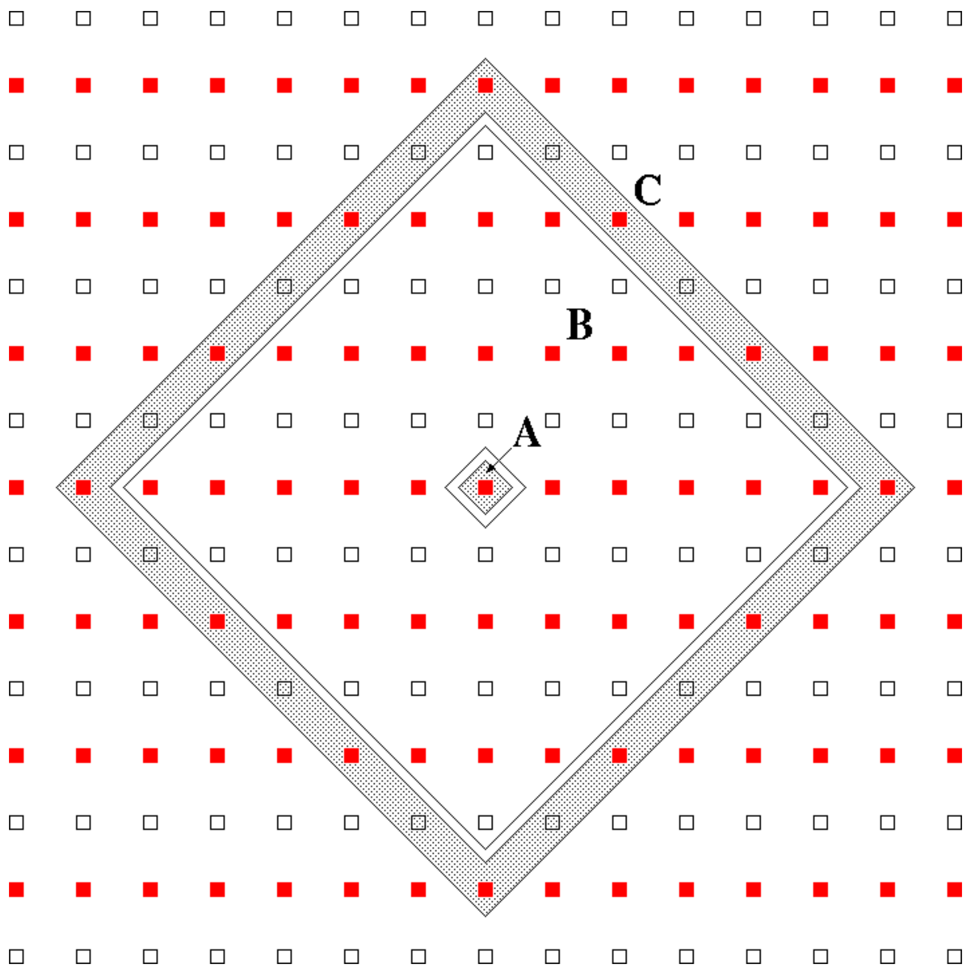
- Full Manhattan circle on a half occupied cell rows gives

$$N_{\mathrm{A}} = 1$$

$$N_{\mathrm{B}} = \ell(\ell - 1)$$

$$N_{\mathrm{C}} = 2\ell$$

corresponding to $\zeta = 0.5$



Flat WLD model

$$w(\ell) = Kq(\ell)D(\ell)$$

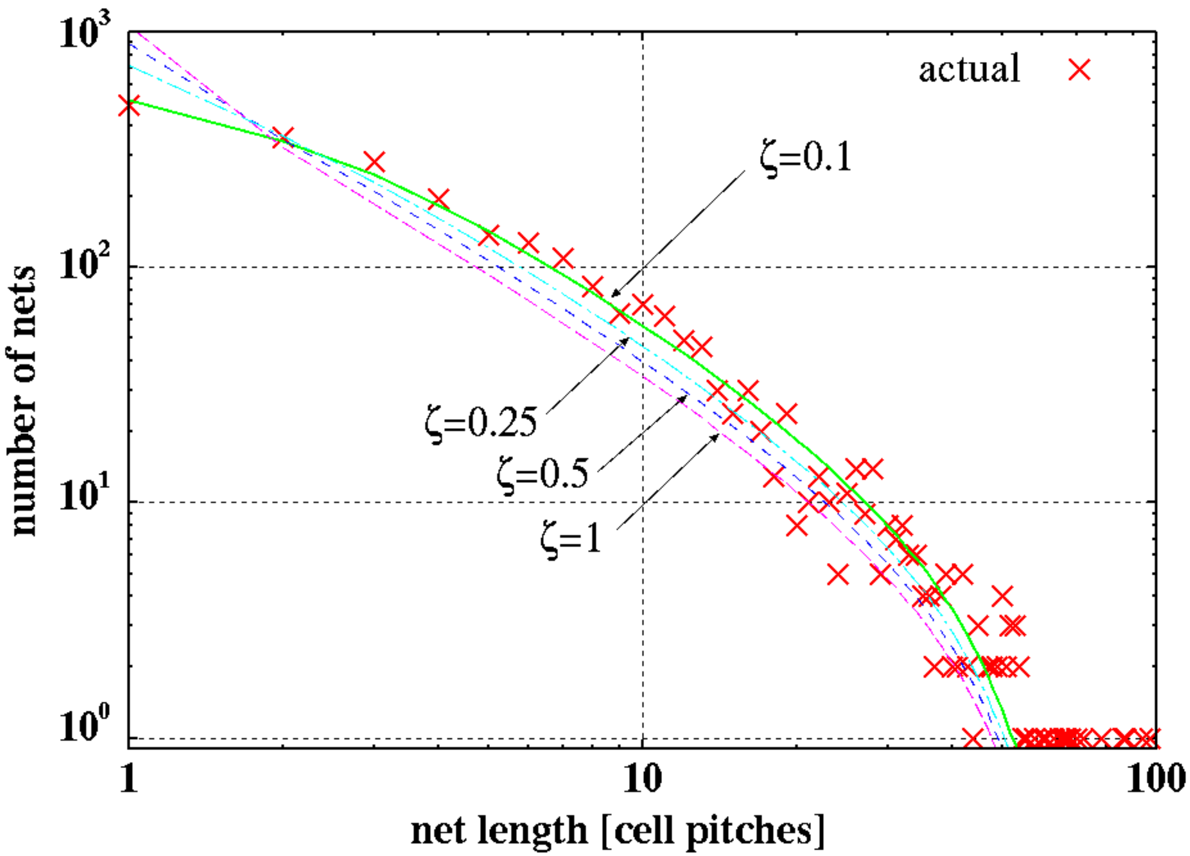
$$K = \frac{W_{\text{total}}}{\sum_{\ell=1}^{\ell_{\max}} q(\ell)D(\ell)}$$

$$q(\ell) = \frac{1}{2\ell}\{[1+2\zeta\ell(\ell-1)]^p + [2\zeta\ell(\ell+1)]^p - [2\zeta\ell(\ell-1)]^p - [1+2\zeta\ell(\ell+1)]^p\}$$

$$D(\ell) = \begin{cases} \dfrac{\ell(6L^2 - 6\ell L + \ell^2 - 1)}{3} & (1 \le \ell < L) \\ \dfrac{(2L-\ell+1)(2L-\ell)(2L-\ell-1)}{3} & (L \le \ell \le 2L-2) \\ 0 & \text{otherwise} \end{cases}$$

Comparison with real data

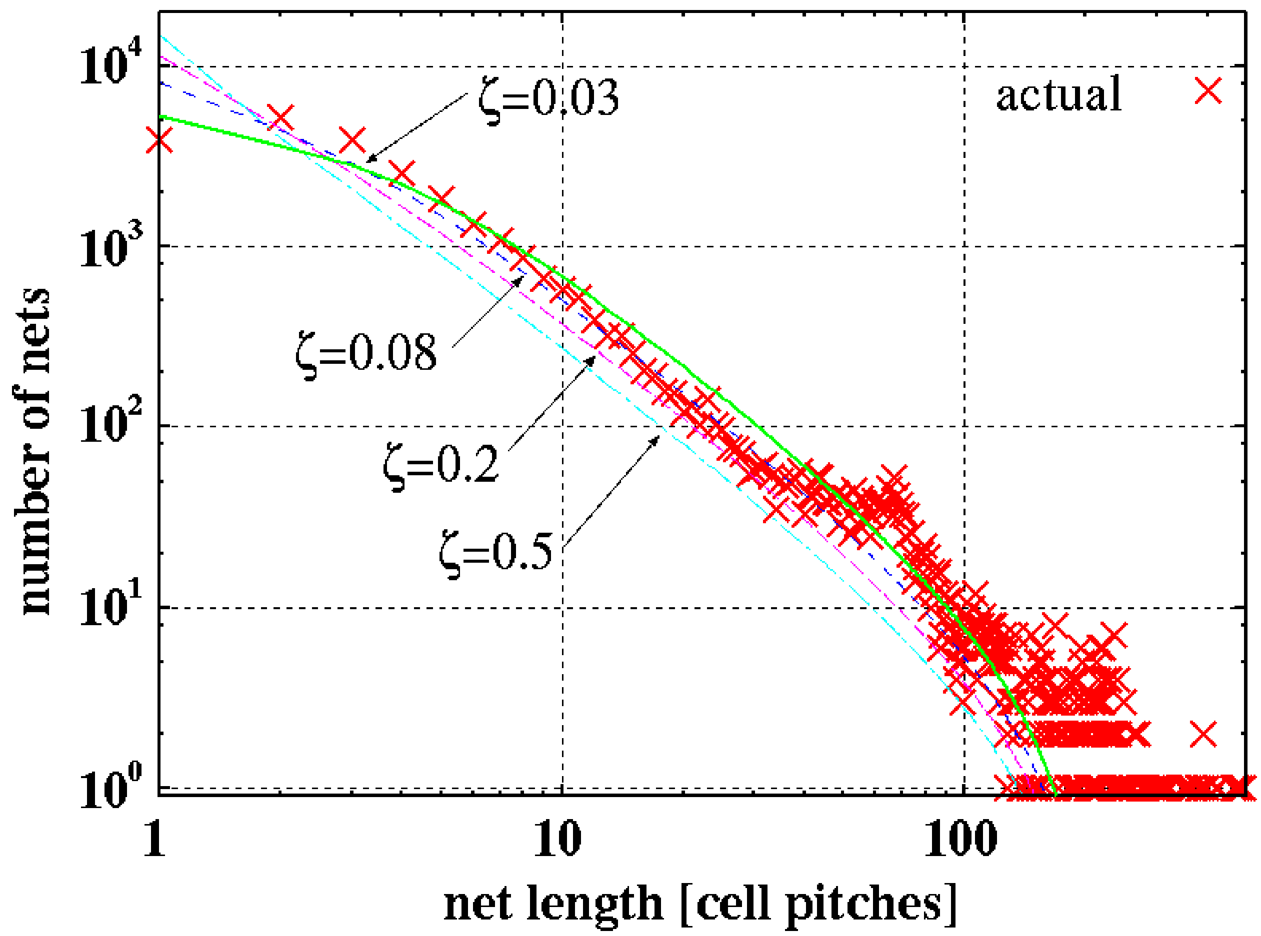
- 180nm CMOS, 2283 cells



Condition	Avg. length
Actual	8.0
ζ= 0.1	7.4
ζ= 0.25	6.2
ζ= 0.5	5.6
ζ= 1	5.0

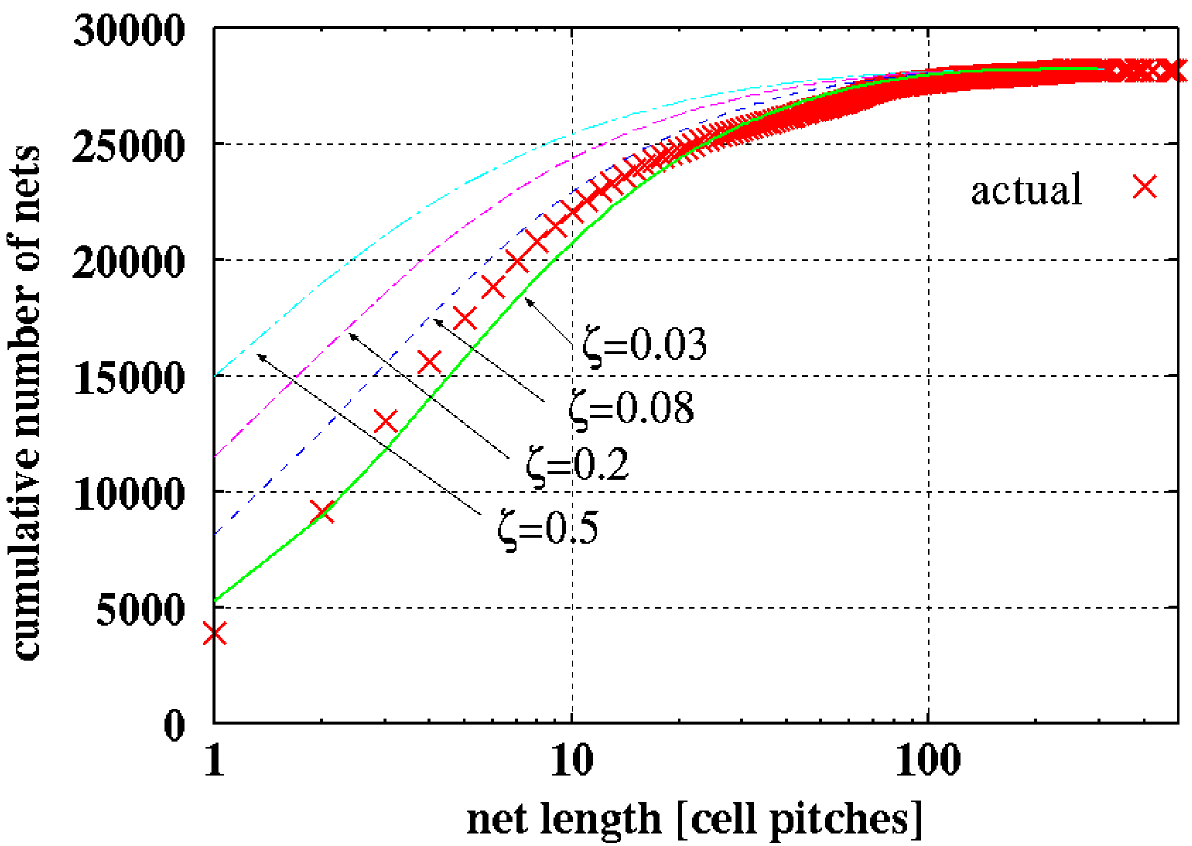
Comparison with real data

- 180nm CMOS, 24868 cells



Comparison with real data

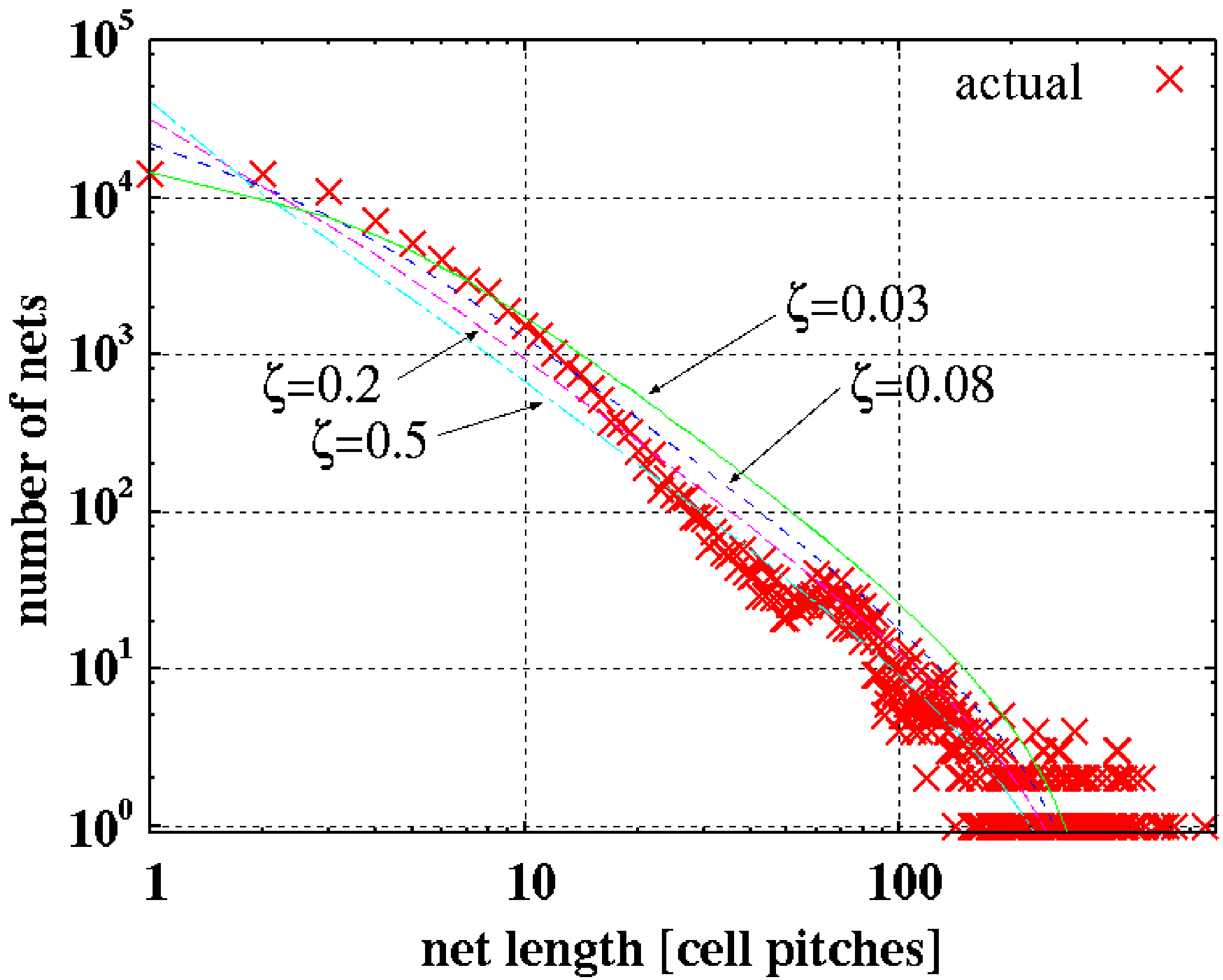
- 180nm CMOS, 24868 cells



Condition	Avg. length
Actual	11.9
ζ= 0.03	10.9
ζ= 0.08	8.2
ζ= 0.2	6.4
ζ= 0.5	5.0

Comparison with real data

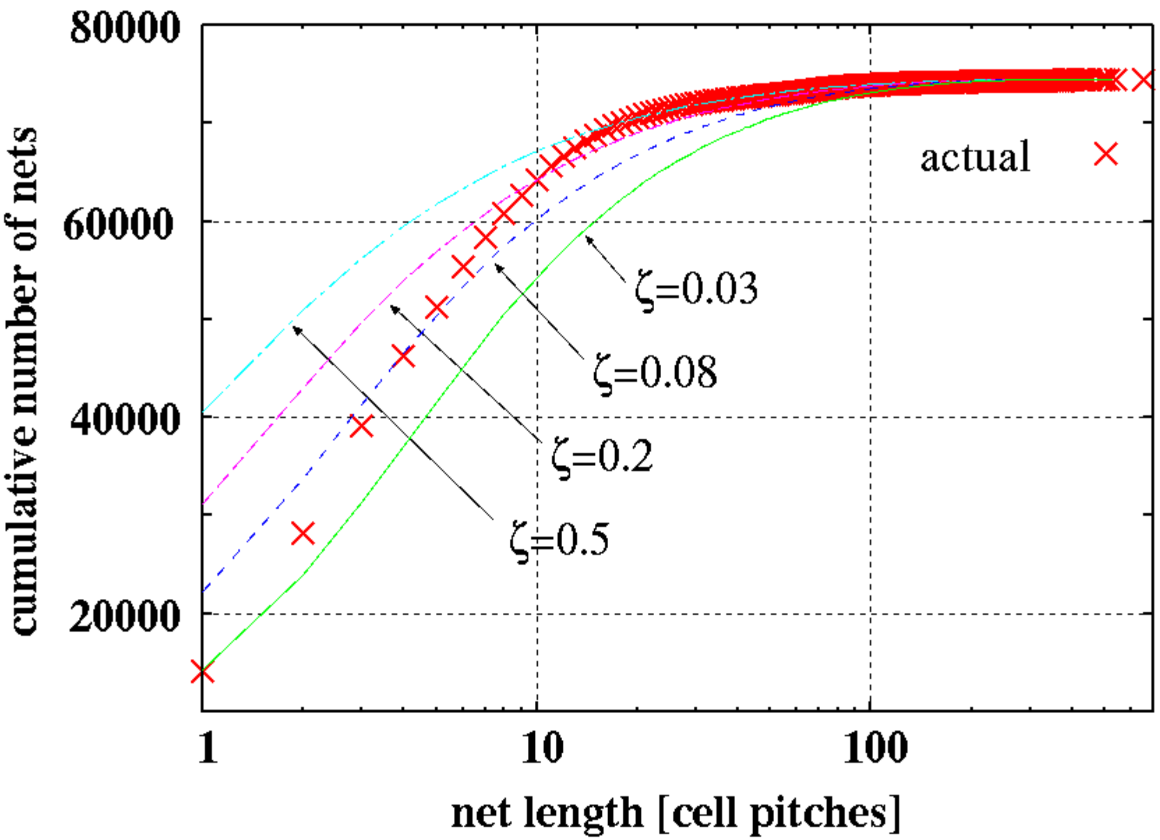
- 130nm CMOS, 70370 cells



Comparison with real data

TOKYO TECH
Pursuing Excellence

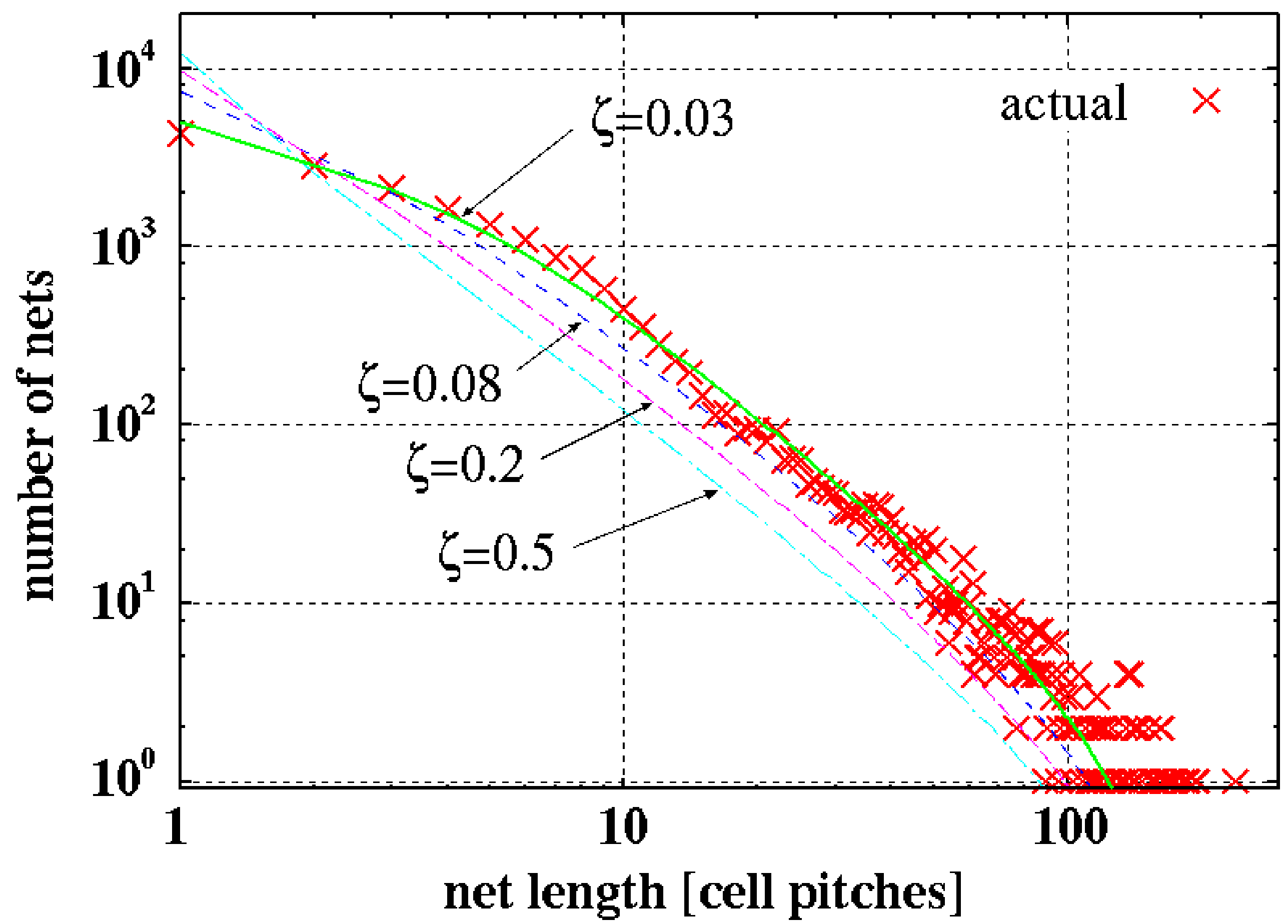
- 130nm CMOS, 70370 cells



Condition	Avg. length
Actual	7.4
ζ= 0.03	12.6
ζ= 0.08	9.4
ζ= 0.2	7.2
ζ= 0.5	5.5

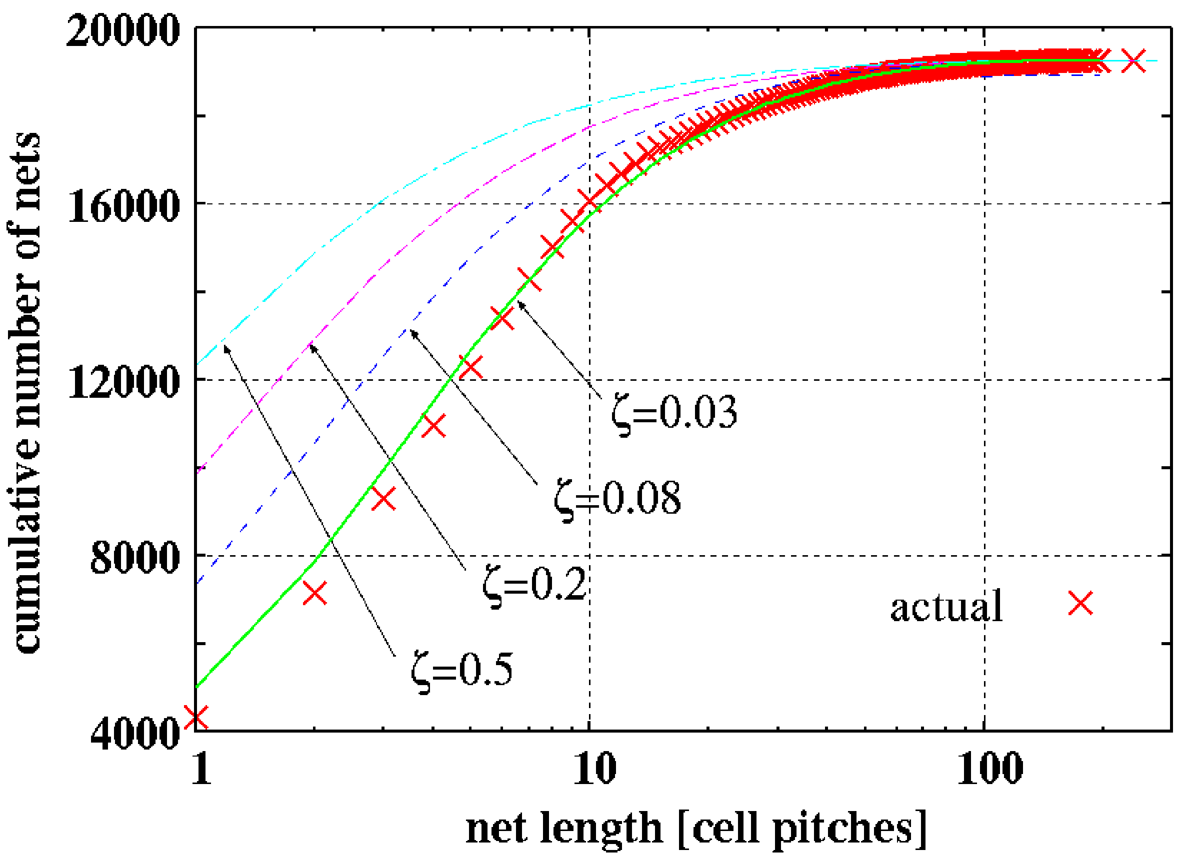
Comparison with real data

- 90nm CMOS, 19321 cells



Comparison with real data

- 90nm CMOS, 19321 cells



Condition	Avg. length
Actual	7.3
ζ= 0.03	7.6
ζ= 0.08	5.5
ζ= 0.2	4.1
ζ= 0.5	3.1

Conclusions

- A simple occupation probability function $q(\ell)$ with an extra degree of freedom that expresses non-idealities is proposed
- The proposed wire-length distribution model better reproduces experimental data especially in the short-wire-length region
- Effort should be made to strengthen the foundations of the model so that the parameter ζ can be determined a priori

Thank you.

TOKYO TECH

Pursuing Excellence

Tokyo Institute of Technology –TOKYO TECH– develops distinctive students with outstanding qualities of creativity and leadership. TOKYO TECH is making significant contributions to science and technology in many fields of expertise, creating new and powerful synergies. TOKYO TECH, being a research-based university, is dedicated to education and research, and to exploring knowledge in science and technology. Pursuing excellence, TOKYO TECH serves society and the world.

$$N_{\mathrm{A}} = 1,\ N_{\mathrm{B}} = 2\ell(\ell - 1),\ N_{\mathrm{C}} = 4\ell$$

$$N_{\mathrm{A}} = 1,\ N_{\mathrm{B}} = \ell(\ell - 1),\ N_{\mathrm{C}} = 2\ell$$

$$N_{\mathrm{A}} = 1,\ N_{\mathrm{B}} = \frac{\ell(\ell - 1)}{2},\ N_{\mathrm{C}} = \ell$$

$$\lim_{\ell \to \infty} \sum_{r=1}^{\ell-1} \mathrm{prob}_{\mathrm{A}\leftrightarrow\mathrm{C}}(r) = 1$$

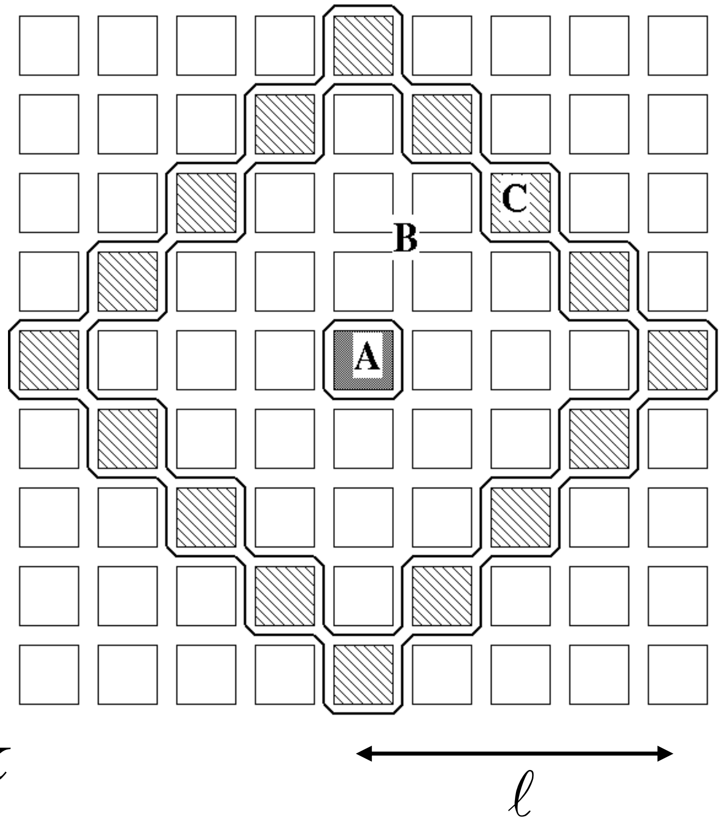
[1] Christie & Stroobandt, "The interpretation and application of Rent's rule," IEEE Trans. VLSI Syst., vol.8, no.6, pp.639-648, 2000.

[2] Davis, De, Meindl, "A stochastic wire-length distribution for gigascale integration (GSI)---Part I: Derivation and validation," IEEE Trans. Electron Devices, vol.45, no.3, pp.580-589, 1998.

Introducing probability

- If blocks A, B, and C are arranged as shown,

$$N_{\mathrm{A}} = 1, N_{\mathrm{B}} = 2\ell(\ell-1), N_{\mathrm{C}} = 4\ell$$



- From the recurrence formula

$$T_{\mathrm{A}\leftrightarrow\mathrm{B}}(\ell+1) = T_{\mathrm{A}\leftrightarrow\mathrm{B}}(\ell) + T_{\mathrm{A}\leftrightarrow\mathrm{C}}(\ell)$$

and Rent's rule follows

$$\lim_{\ell\to\infty} T_{\mathrm{A}\leftrightarrow\mathrm{B}}(\ell) = \lim_{\ell\to\infty} \sum_{r=1}^{\ell-1} T_{\mathrm{A}\leftrightarrow\mathrm{C}}(r) = k$$

- A probability function can be defined by

$$\mathrm{prob}_{\mathrm{A}\leftrightarrow\mathrm{C}}(\ell) = \frac{1}{k} T_{\mathrm{A}\leftrightarrow\mathrm{C}}(\ell)$$

Occupation probability

- We want the occupation probability $q(\ell)$ to satisfy

$$k = \alpha k \sum_{\ell=1}^{\infty} 4\ell q(\ell) \approx \alpha k \int_0^{\infty} \frac{4\ell}{c_\mathrm{p}^2} q(\ell)\mathrm{d}\ell = \alpha \int q(\ell) k \frac{\mathrm{d}S}{c_\mathrm{p}^2}$$

k : Average number of terminals per cell

c_p : Cell pitch, $S = 2\ell^2$: Area of Manhattan circle

α : Terminal-to-wire conversion factor

$$\Rightarrow \quad q(\ell) = \frac{1}{2\ell} \mathrm{prob}_{\mathrm{A}\leftrightarrow\mathrm{C}}(\ell)$$