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Statistical Crosstalk Aggressor Alignment Aware Interconnect Delay Calculation

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Outline

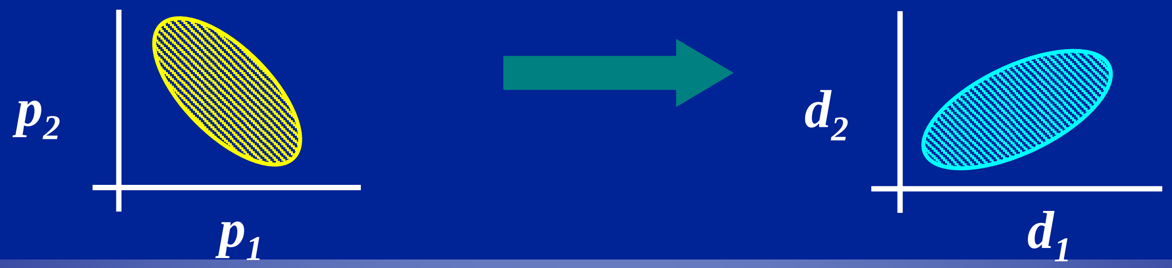
- SSTA Background
- Problem formulation
- Method: theory and implementation
- Experiment
- Summary

Background: Variability

- Increased variability in nanometer-scale VLSI designs
  - **Process:**
    - OPC → Lgate
    - CMP → thickness
    - Doping → Vth
  - **Environment:**
    - Supply voltage → transistor performance
    - Temperature → carrier mobility μ and Vth
- These (PVT) variations result in circuit performance variation

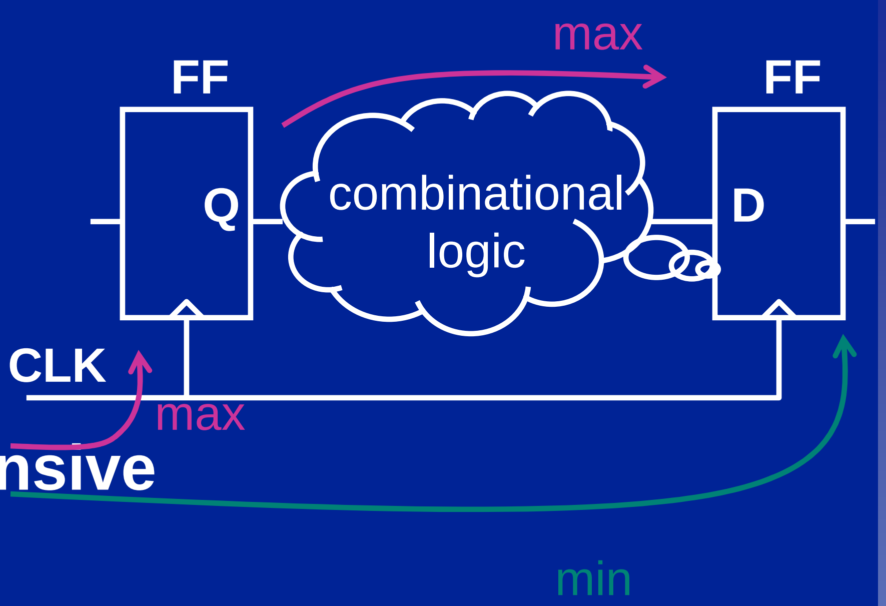
PVT Parameter Distributions

Gate/net Delay Distribution



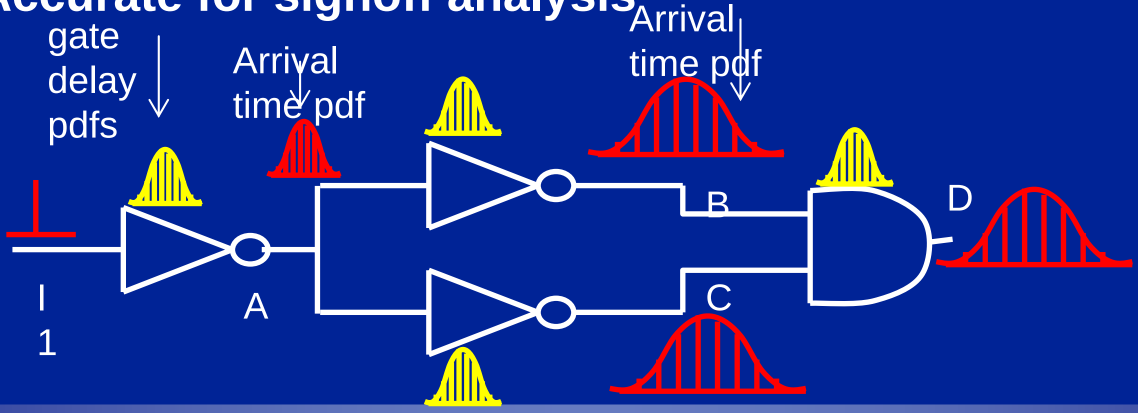
Background: Timing Analysis

- Min/Max-based
  - Inter-die variation
  - Pessimistic
- Corner-based
  - Intra-die variation
  - Computationally expensive
- Statistical
  - PDF for delays
  - Reports timing yield



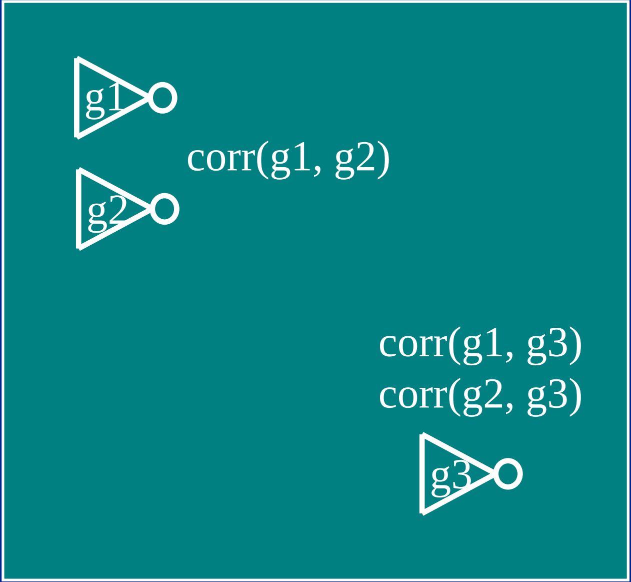
Background: SSTA

- **Represent delays and signal arrival times as random variables**
  - ## Block-based
    - **Each timing node has an arrival time distribution**
    - **Static worst case analysis**
    - **Efficient for circuit optimization**
  - ## Path-based
    - **Each timing node *for each path* has an arrival time distribution**
    - **Corner-based or Monte Carlo analysis**
    - **Accurate for signoff analysis**



Background: SSTA Correlations

- Delays and signal arrival times are random variables
- Correlations come from
  - Spatial
    - inter-chip, intra-chip, random variations
  - Re-convergent fanout
    - Multiple-input switching
    - Cross-coupling
    - ……



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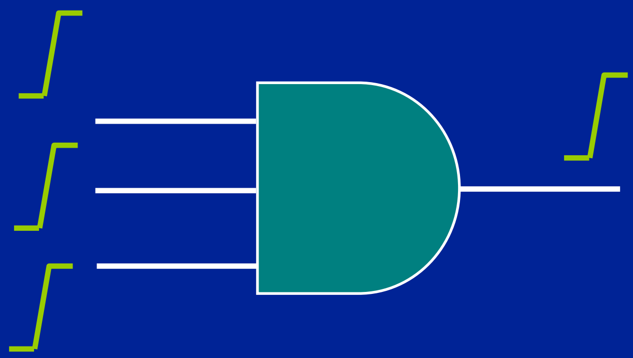
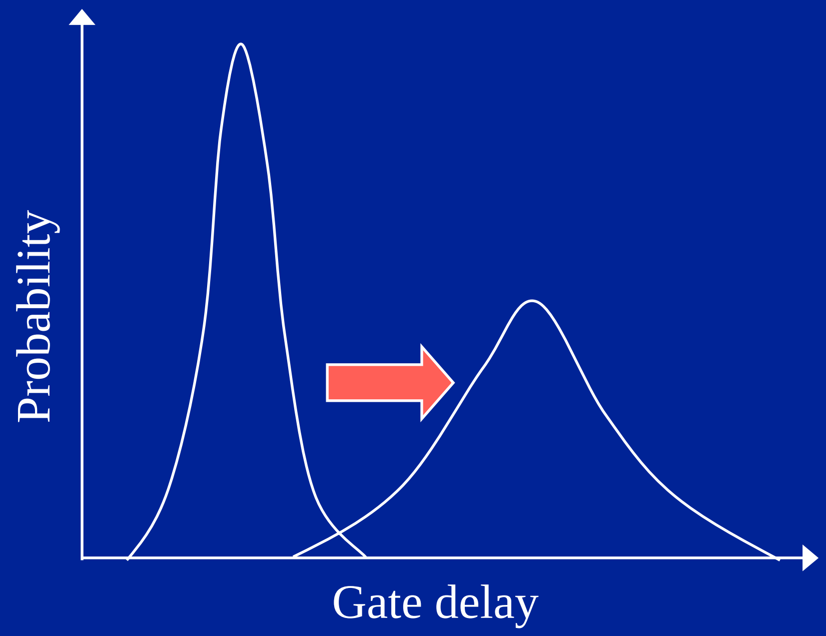
Multiple-Input Switching

- Simultaneous signal switching at multiple inputs of a gate leads to up to 20%(26%) gate delay mean (standard deviation) mismatch
  - [Agarwal-Dartu-Blaauw, DAC-04]





Crosstalk Aggressor Alignment

- Crosstalk aggressor alignment induced interconnect delay variation is an equally significant source of uncertainty in SSTA

Problem Formulation

- Given
  - Coupled interconnect system
  - Input signal arrival time distributions
- Find
  - Output signal arrival time distributions
- We represent signal arrival times as polynomial functions of normally distributed random variables

$$x_i = f_i(r_1, r_2, \ldots)$$
$$r_i \sim N(\mu_i, \sigma_i)$$

- E.g., for first order approximation of two input signal arrival times, their skew (crosstalk alignment) is given by normally distributed random variables with correlation taken into account

$$x_1 \sim N(\mu_1, \sigma_1) \qquad x_2 \sim N(\mu_2, \sigma_2)$$
$$x' = x_2 - x_1 \sim N(\mu' = \mu_2 - \mu_1, \sigma' = (\sigma_1^2 + \sigma_2^2 + corr)^{1/2})$$

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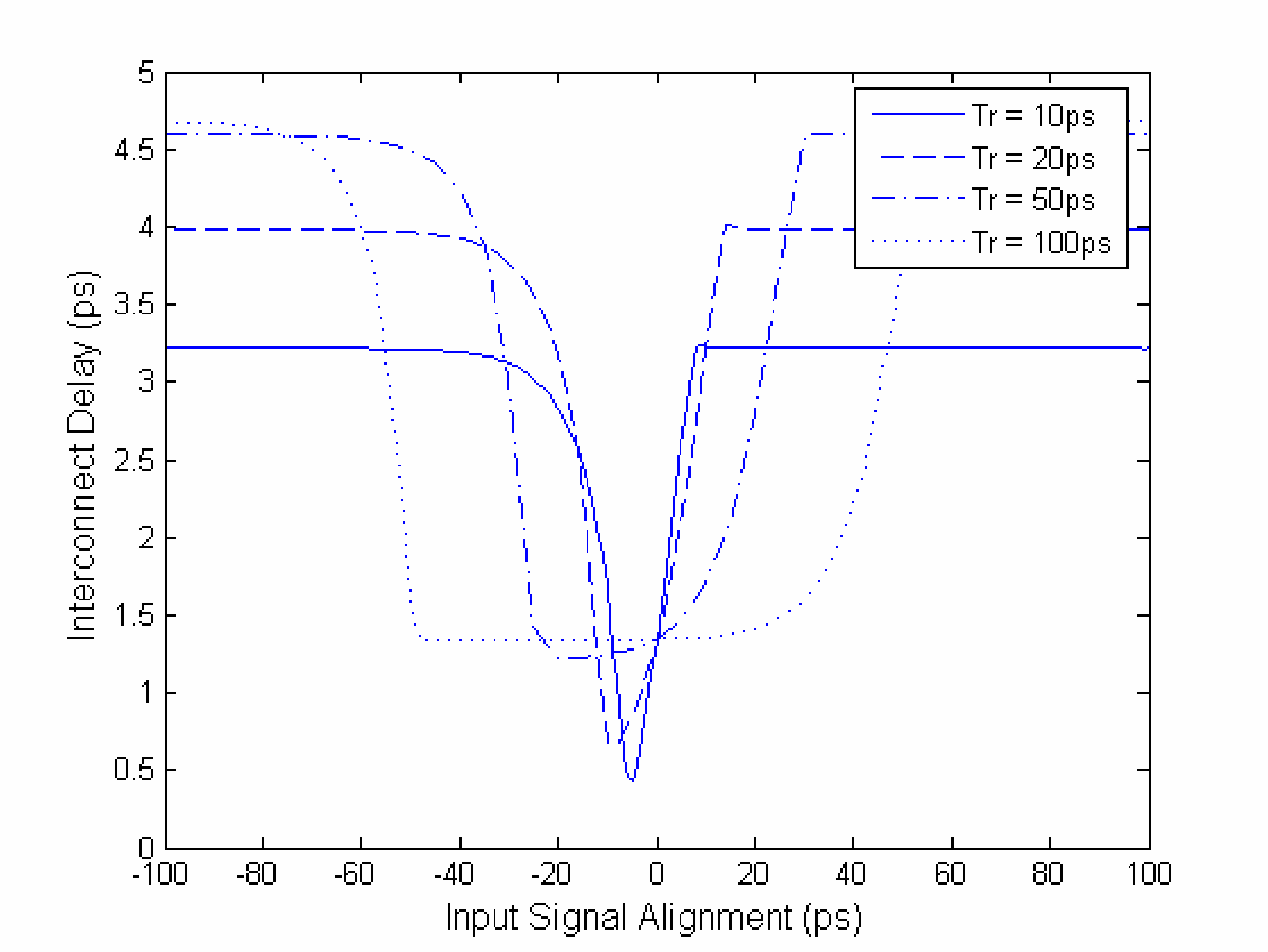
Interconnect Delay as a Function of Crosstalk Alignment

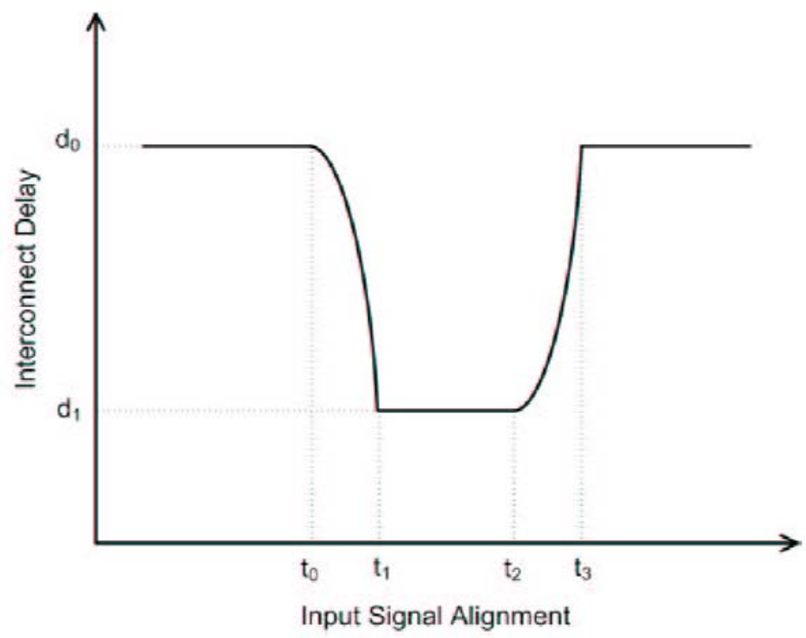


For 1000um global interconnects in 90nm technology

Interconnect Delay as a Function of Crosstalk Alignment

- More complex than the timing window concept
- Can be computed by simulation or delay calculation
- Approximate as a piecewise quadratic function

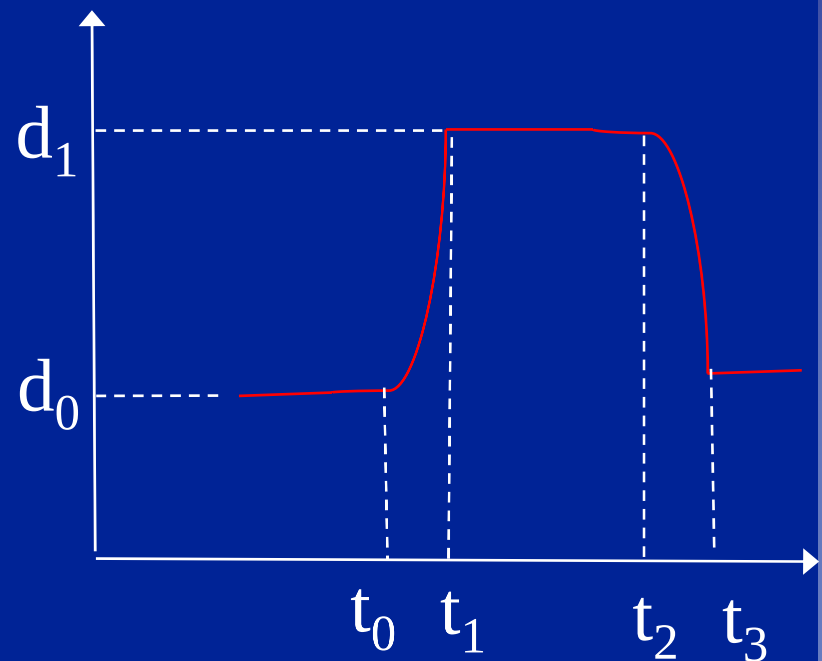
$$\tau = \begin{cases} d_0 & x' \le t_0 \\ a_0 + a_1 x' + a_2 x'^2 & t_0 \le x' \le t_1 \\ d_1 & t_1 \le x' \le t_2 \\ b_0 + b_1 x' + b_2 x'^2 & t_2 \le x' \le t_3 \\ d_0 & t_3 \le x' \end{cases}$$



Interconnect Delay as a Function of Crosstalk Alignment

- Similarly, we can model the case of input signals switching in opposite directions
  - (not discussed here)

$$\tau = \begin{cases} d_0 & x' \le t_0 \\ a_0 + a_1 x' + a_2 x'^2 & t_0 \le x' \le t_1 \\ d_1 & t_1 \le x' \le t_2 \\ b_0 + b_1 x' + b_2 x'^2 & t_2 \le x' \le t_3 \\ d_0 & t_3 \le x' \end{cases}$$



Closed-Form Interconnect Delay Distribution

For a normal distribution of crosstalk alignment

$$P(\tau)=\begin{cases}\frac{1}{2}\left(erf\left(\frac{t_2-\mu'}{\sigma'}\right)-erf\left(\frac{t_1-\mu'}{\sigma'}\right)\right) & \tau=d_1\\ -\frac{1}{\sqrt{2\pi}\sigma'}\left(\frac{1}{\sigma_a}e^{-\frac{(-\sigma_a-a_1-2a_2\mu')^2}{8a_2^2\sigma'^2}}+\frac{1}{\sigma_b}e^{-\frac{(-\sigma_b-b_1-2b_2\mu')^2}{8b_2^2\sigma'^2}}\right) & d_1\le\tau\le d_0\\ 1-\frac{1}{2}\left(erf\left(\frac{t_3-\mu'}{\sigma'}\right)-erf\left(\frac{t_0-\mu'}{\sigma'}\right)\right) & \tau=d_0\end{cases}$$

$$\sigma_a=\sqrt{4a_2(\tau-a_0)+a_1^2}$$

$$\sigma_b=\sqrt{4b_2(\tau-b_0)+b_1^2}$$

Closed-Form Interconnect Output Signal Arrival Time Distribution

For a normal distribution of crosstalk alignment

$$P(y)= \frac{1}{2\sqrt{2\pi}\sigma_{ya}} e^{-\frac{(y-\mu_{ya})^2}{2\sigma_{ya}}} \left(F(y,t_1,a_0,a_1,\sigma_{ya})-F(y,t_0,a_0,a_1,\sigma_{ya})\right)+$$

$$\frac{1}{2\sqrt{2\pi}\sigma_{yb}} e^{-\frac{(y-\mu_{yb})^2}{2\sigma_{yb}}} \left(F(y,t_1,b_0,b_1,\sigma_{yb})-F(y,t_0,b_0,b_1,\sigma_{yb})\right)+$$

$$\frac{P_1(y_1-d_0)}{2}\left(2-Erf\left(\frac{t_3-\mu_2+y-d_0}{\sqrt{2}\sigma'}\right)+Erf\left(\frac{t_0-\mu_2+y-d_0}{\sqrt{2}\sigma'}\right)\right)+$$

$$\frac{P_1(y_1-d_1)}{2}\left(Erf\left(\frac{t_3-\mu_2+y-d_1}{\sqrt{2}\sigma'}\right)-Erf\left(\frac{t_0-\mu_2+y-d_1}{\sqrt{2}\sigma'}\right)\right)$$

$$\mu_{ya}=\mu_1+a_0-a_1(\mu_1-\mu_2) \quad \sigma_{ya}=\sqrt{(1-a_1^2)\sigma_1^2+a_1^2\sigma'^2}$$

$$\mu_{yb}=\mu_1+b_0-b_1(\mu_1-\mu_2) \quad \sigma_{yb}=\sqrt{(1-b_1^2)\sigma_1^2+b_1^2\sigma'^2}$$

Statistical Delay Calculation for Coupled Interconnects

Input: Coupled interconnects
Input signal arrival time distributions
Process variations

Output: Output signal arrival time distributions

1. Interconnect delay calculation for sampled crosstalk alignments
2. Approximate interconnect delay in a piece-wise quadratic function of crosstalk alignment
3. Compute output signal arrival time distribution by closed-form formulas
4. Apply superposition for each input
5. Combine with other process variations

Multiple Aggressors / Variations

- An RLC interconnect system is a *linear* system, which enables us to apply superposition to model the effects of multiple crosstalk aggressors
- Correlated variation sources
  - For each condition
    - Compute *conditional probabilities*
  - Combine conditional probabilities
- Independent variation sources
  - *Superposition*

Runtime Analysis

- ***Interconnect delay calculation for N sampled crosstalk alignment takes O(N) time***
  where N = min(1.5 * max input transition time, 6 * sigma of crosstalk alignment) / time_step
- **Fitting takes O(N) time**
- **Computing output signal arrival time distribution takes constant time, e.g., when updating statistical delay calculation in an iterative SSTA**

Iterative SSTA

- **STA-SI goes through an *iteration of timing window refinement for reduced pessimism* of worst case analysis**
- **SSTA-SI goes through an *iteration of signal arrival time pdf refinement with reduced deviations***

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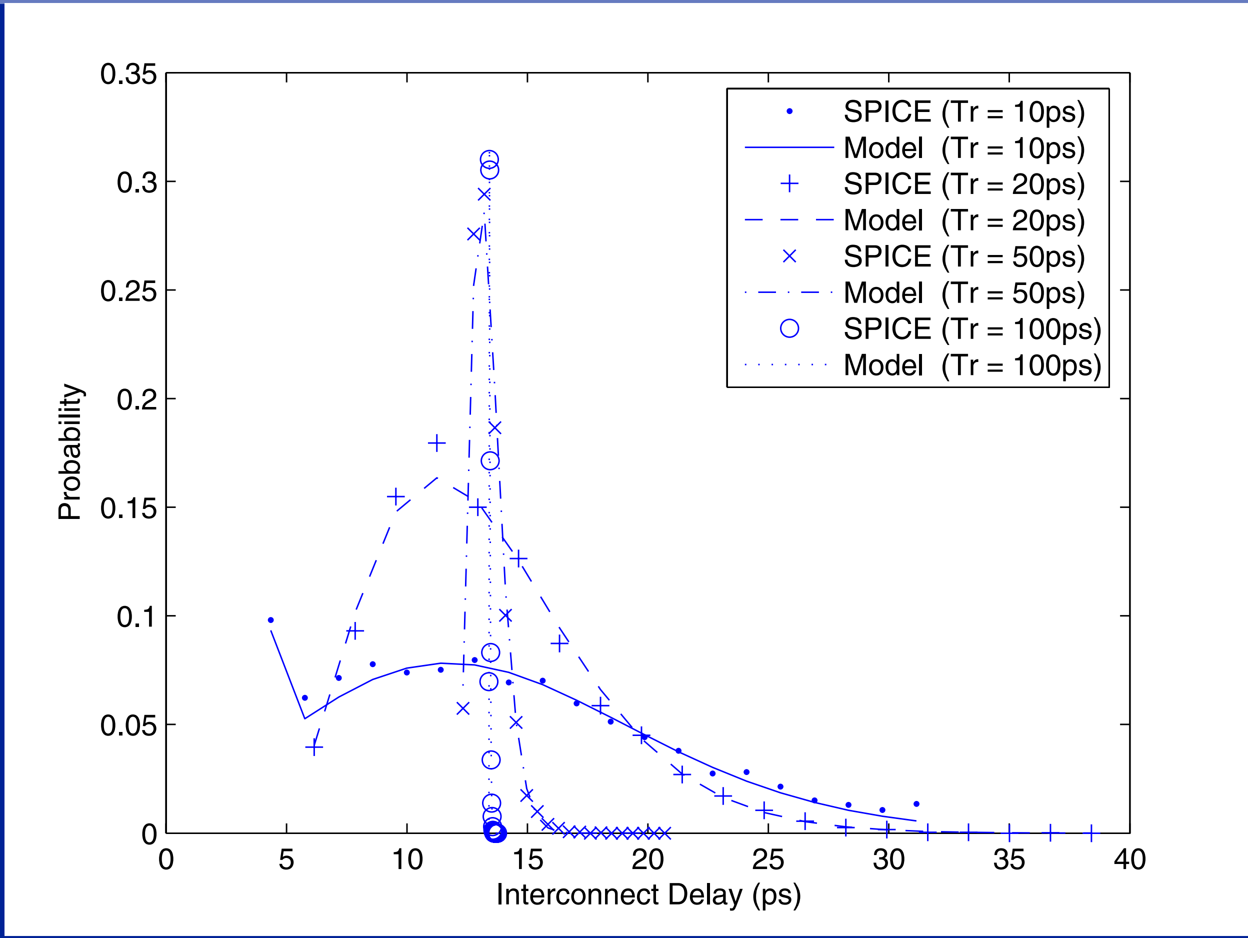
Experimental Setting

- Quadratic function regression performed with Origin
- Monte Carlo simulation performed with HSPICE
- Closed-form model-based distribution calculation: C
- Testcases:
  - 70nm BTPM model (Cao et al.)
  - 130nm industry designs

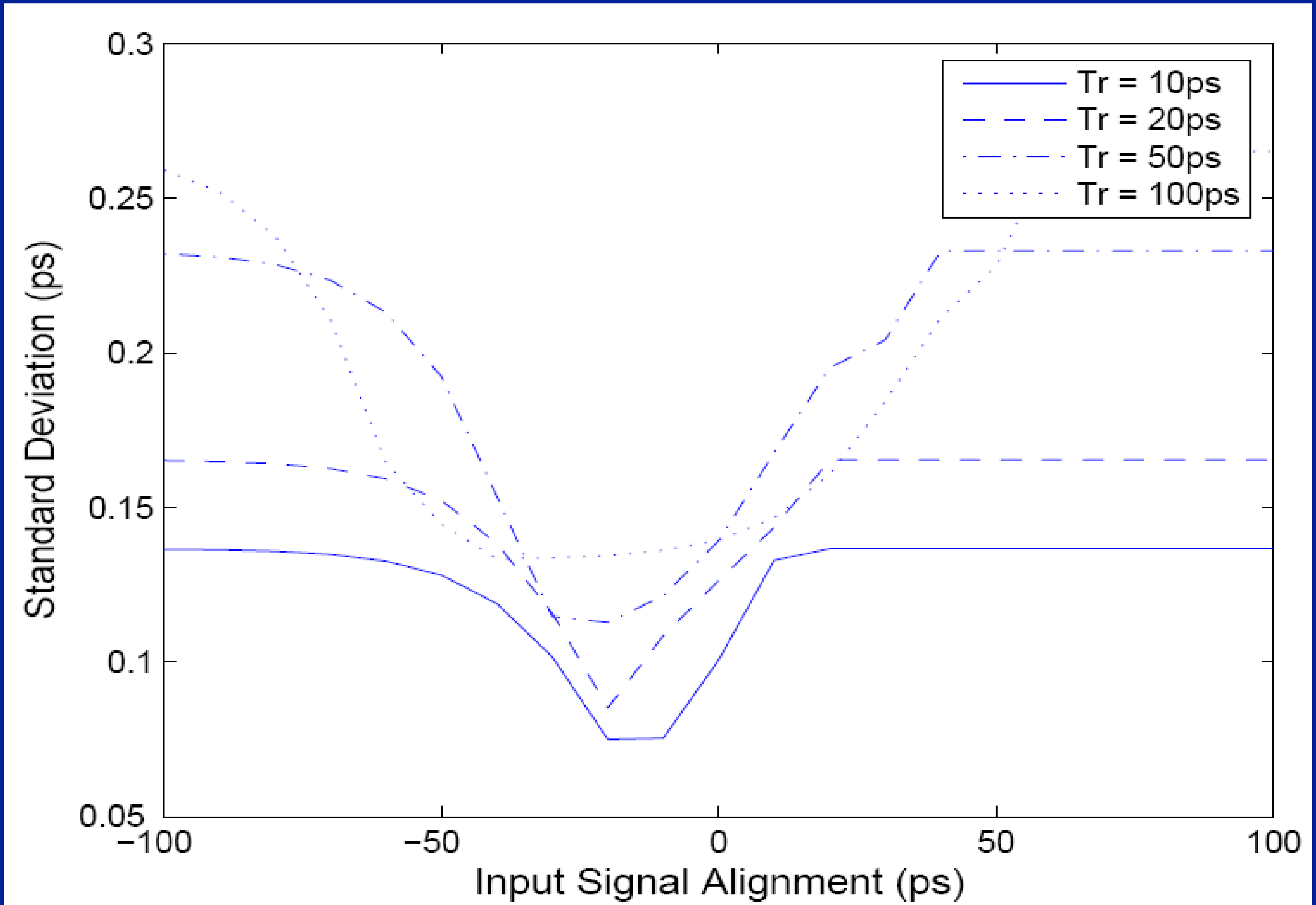
70nm	L (um)	W(um)	S(um)	T(um)
global	1000	0.45	0.45	1.20
intermediate	200	0.14	0.14	0.35
local	30	0.10	0.10	0.20

Interconnect Delay Distribution



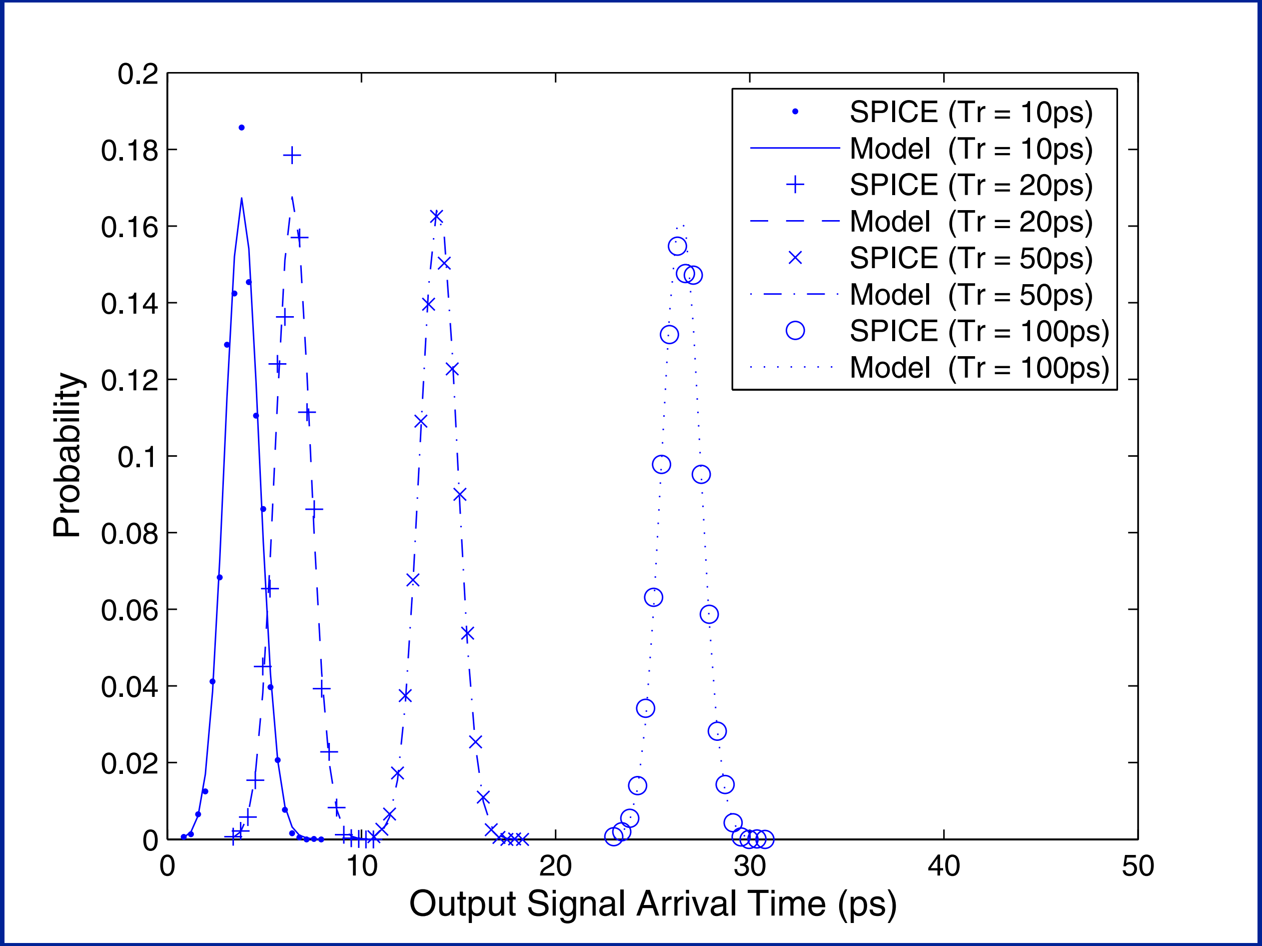
For a pair of 1000um coupled global interconnects in 70nm BPTM technology, with 10, 20, 50 and 100ps input signal transition time, and crosstalk alignment in a normal distribution N(0, 10ps)

Interconnect Delay Standard Deviation due to Varied Wire Width



For a pair of 1000um coupled global interconnects in 70nm BPTM technology, with 10, 20, 50 and 100ps input signal transition time, and wire width variation in a normal distribution N(0, 10%)

Interconnect Output Signal Arrival Time Distribution



For a pair of 1000μm coupled global interconnects in 70nm BPTM technology, with 10, 20, 50 and 100ps input signal transition time, and crosstalk alignment in a normal distribution N(0, 6ps)

Experiment

Test case 1: 1000μm interconnects of 70nm BPTM technology

		Delay				Output					
		SPICE		Model		SPICE		Model		% diff	
3σ	Tr(ps)	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
50	50	3.83	0.85	3.82	0.83	29.4	16.2	29.7	16.6	0.78	2.46
100	100	3.82	0.92	3.84	0.83	54.8	32.8	55.6	33.9	1.52	3.38
200	200	3.78	0.96	3.78	0.82	105.2	65.9	106.3	67.0	1.06	1.65

Test case 2: interconnects in a 130μm industry design

		Delay				Output					
		SPICE		Model		SPICE		Model		% diff	
3σ	Tr(ps)	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
50	100	4.29	0.16	4.30	0.15	54.5	16.4	53.4	16.1	–2.09	-0.05
100	100	4.30	0.18	4.30	0.17	54.8	32.9	54.1	33.0	-0.17	0.18
200	200	4.25	0.18	4.25	0.16	105.2	65.9	104.9	66.1	-0.28	0.35

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Summary

- ***SSTA must consider SI effects!***
- We take crosstalk aggressor alignment into account in statistical interconnect delay calculation
  - We approximate interconnect delay as ***a piecewise quadratic function*** of crosstalk aggressor alignment
  - We derive ***closed-form formulas*** for interconnect delay and output signal arrival time distribution for given input signal arrival times in polynomial functions of normal distributions
  - Our experiments show that neglecting crosstalk alignment effect could lead to up to ***114.65% (71.26%)*** mismatch of interconnect delay means (standard deviations), while our method gives output signal arrival time means (standard deviations) within ***2.09% (3.38%)*** of SPICE results

Thank you !

Experiment: Delay Variation

	SPICE	Without Crosstalk		Our	
Tr =10ps	μ σ	μ Diff(%)	σ Diff(%)	μ Diff(%)	σ Diff(%)
μ'= -10ps	3.13 0.38	5.65 80.5	0.17 -56	3.12 -0.64	0.38 -0.1
μ'= 0ps	3.38 0.58	5.63 66.5	0.17 -71	3.38 0.0	0.57 -1.9
μ'= 10ps	4.85 0.31	5.64 16.3	0.17 -47	4.85 0.0	4.32 -0.23
Tr =20ps	μ σ	μ Diff(%)	σ Diff(%)	μ Diff(%)	σ Diff(%)
μ'= -10ps	3.14 0.37	6.52 114	0.19 -47	3.12 -0.96	0.37 1.26
μ'= 0ps	3.67 0.47	6.53 77.9	0.19 -58	3.66 -0.01	0.47 -1.0
μ'= 10ps	5.22 0.55	6.52 24.9	0.19 -65	5.22 0.0	0.54 -1.48

Test case: 1000μm interconnects of 70nm BPTM technology

Assume: wire width distribution N(1, 0.05) * Normal_width

crosstalk alignment distribution N (μ', 3.33ps)