# Prediction of Interconnect Net-Degree Distribution Based on Rent's Rule

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# Outline

- Motivation
- Previous work
- Our models:

  - Model 1: Linear

  - Model 2: Exponential

  - Model 3: Weighted exponential
- Experimental results
- Behavior of internal fraction factor *f*
- Results
- Conclusions

# A *priori* Interconnect Prediction

- Interconnect: importance of wires increases (they do not scale as devices do)
  - delay, power, area
- A *priori*:
  - before the actual layout is generated
  - very little information is known
- To narrow the solution search space
- To reduce the number of iterations
- To evaluate new architectures

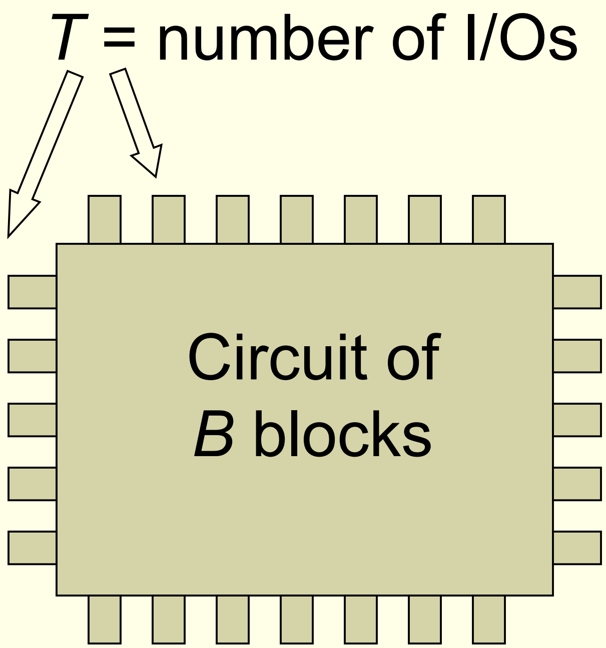
# Rent's Rule

Rent's rule: underlying assumption for most of the interconnect prediction techniques.

$$T = kB^p$$

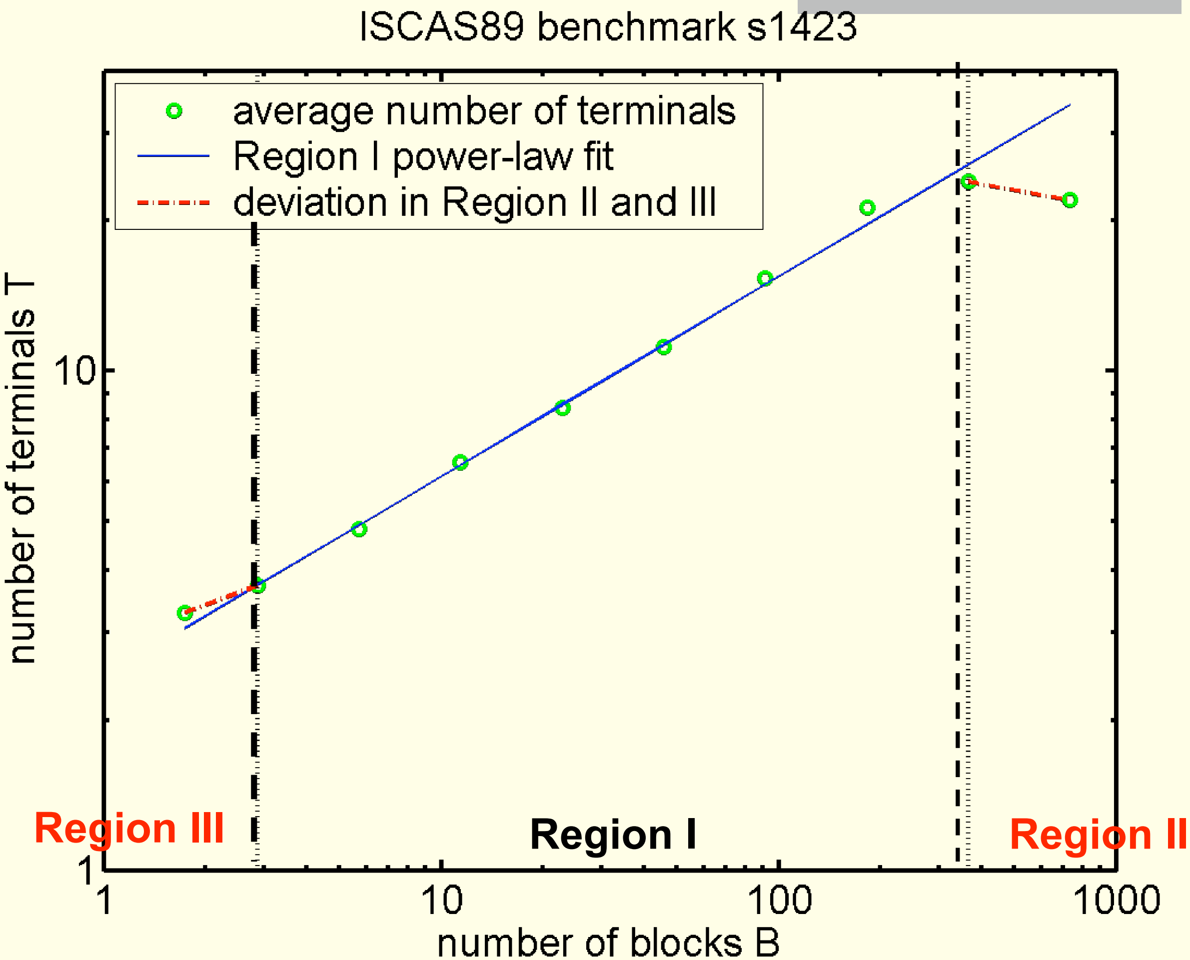
*T*: number of terminals
*B*: number of blocks
*k*: Rent coefficient
- average number of terminals per block

*p*: Rent exponent
- interconnect complexity
- level of placement optimization



# Rent's Rule

- ***Region I:*** **good approximation**
- ***Region II*: high *B***
  - **- technology constraints, limited pins**
  - **- psychological constraints, designers**
- ***Region III*: low *B***
  - **- mismatch between local and global interconnect complexity, FPGAs**

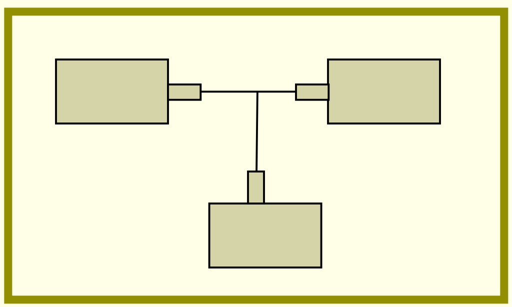


# What is a Net-Degree Distribution?

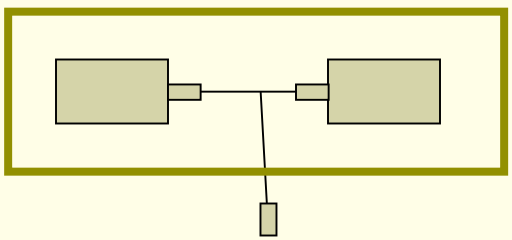
A net-degree distribution is a collection of values, indicating, for each net degree *i*, how many nets have a net degree equal *i* .

**ISCAS89 *s1423* net-degree distribution**

| net degree | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| number of nets | 0 | 568 | 58 | 65 | 23 | 12 | 12 | 2 | 2 | 3 |



**Internal 3-terminal net**



**External 3-terminal net**

# Why Net-Degree Distribution ?

- For a *priori* interconnect prediction, the most important parameter to predict is wire length
- Most current estimation techniques on wire length distribution are based on two-terminal nets

  \- not accurate, lots of wires in current design are multi-terminal nets

  \- multi-terminal nets do change conventional wire length distribution models

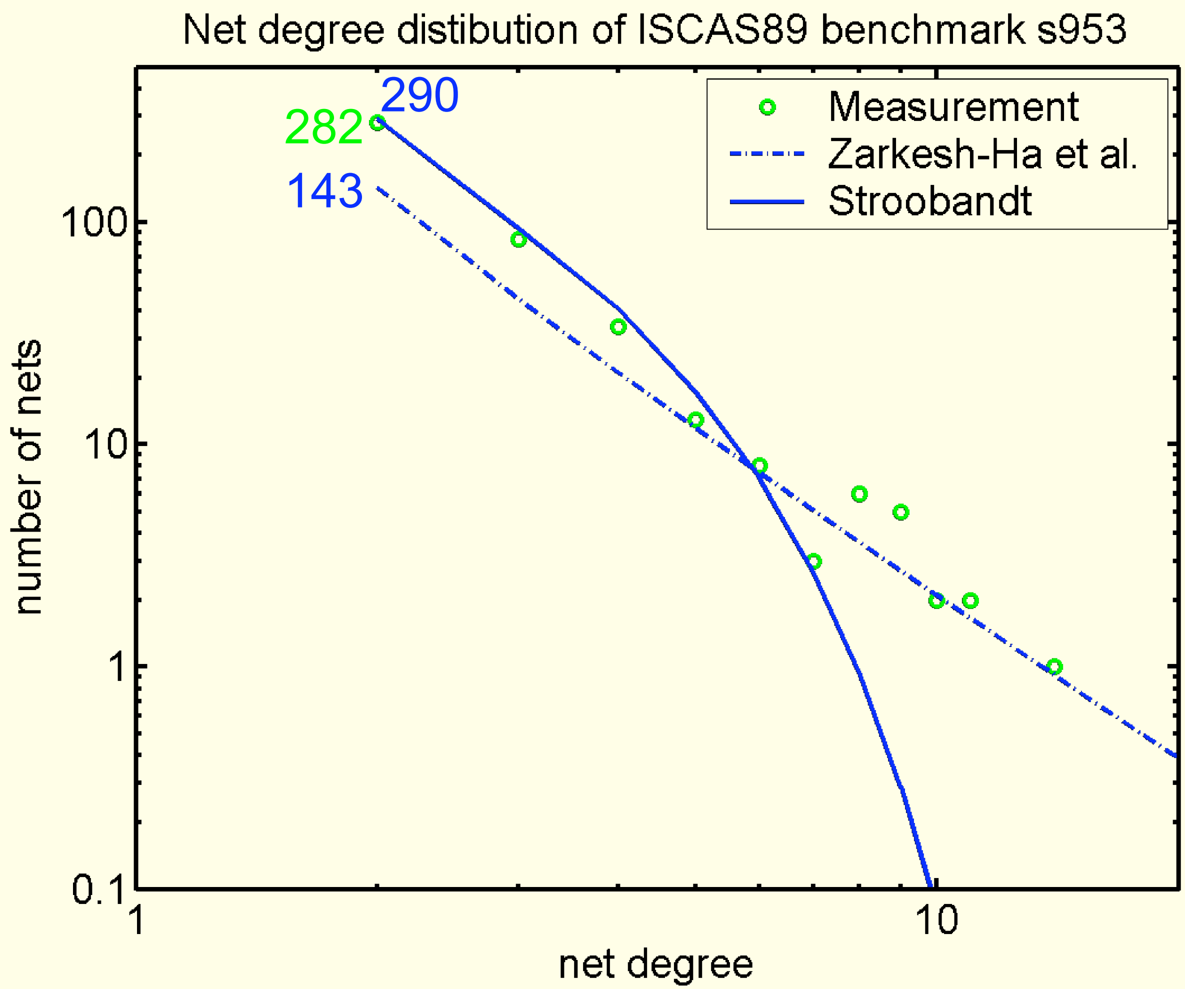
# Why Net-Degree Distribution ?

- To evaluate total wire length - we have to accurately estimate the net-degree distribution
- Current net-degree distribution models not accurate

  - Zarkesh-Ha et al.'s model underestimates the number of nets with small net degrees

  - Stroobandt's model underestimates the number of nets with high net degrees
- The goal of this research was to derive more accurate net-degree distribution models

# Previous Work



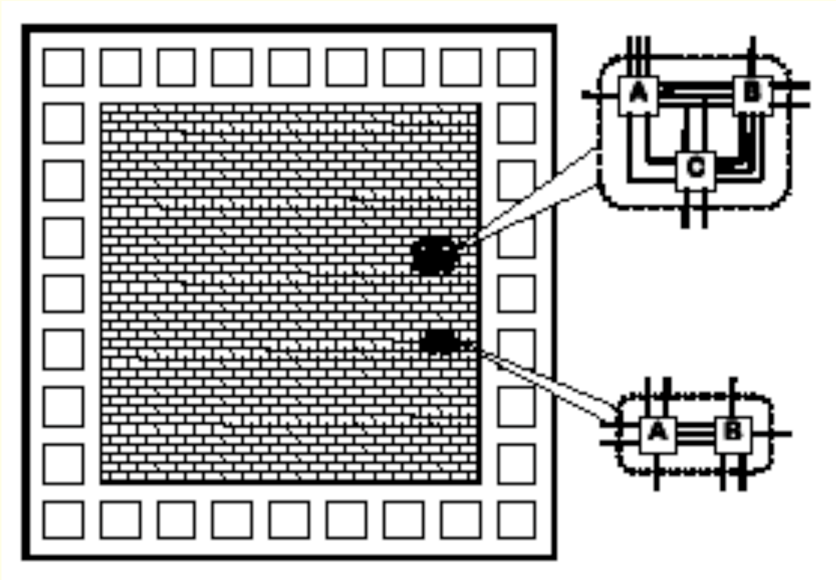
# Zarkesh-Ha et al.'s Model

- A closed form expression for fan-out (net-degree) distribution
- Depends on Rent's parameters *k*, *p* and circuit size *B* only
- Underestimates the distribution for low net degrees, finds inaccurate total number of nets and average net degree

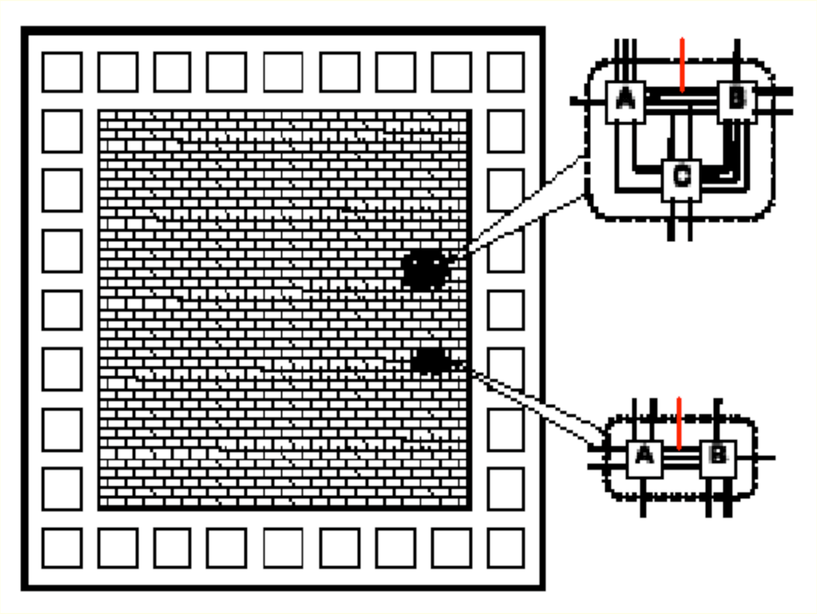
# Zarkesh-Ha et al.'s Model

Zarkesh-Ha et al. made a few inaccurate assumptions

- Did not consider external nets




Their original circuit model

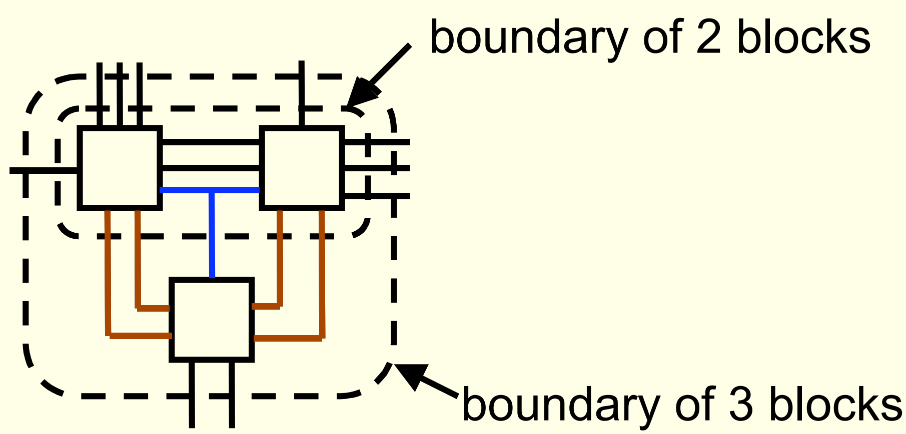



Their circuit model with external nets

# Zarkesh-Ha et al.'s Model

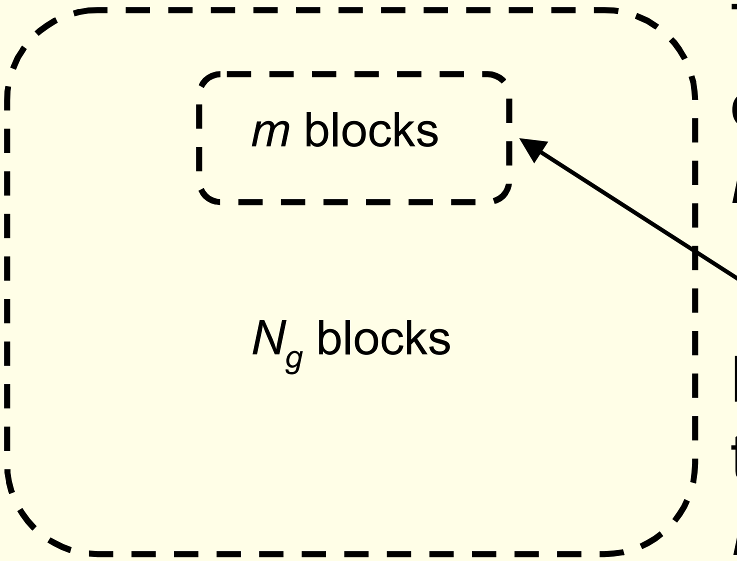
- They assumed that adding one block to the boundary of *m*-1 blocks would only introduce *m*-terminal nets, which is not valid in most circumstances

  - new nets don't have to be connected to all the blocks in the boundary



# Zarkesh-Ha et al.'s Model

- They changed the definition of $T_{Net}(m)$, more specifically, they changed the boundary




They derived the expression of $T_{Net}(m)$ for the boundary of $m$ blocks

$$T_{Net}(m) = k((m-1)^{p-1} - m^{p-1})$$

But when they applied it to the whole circuit consisting of $N_g$ blocks, they forgot to substitute $N_g$ for $m$

# Stroobandt's Model

- Hierarchical model with recursive net-degree distribution
- Depends on Rent's parameters *k*, *p*, circuit size *B,* and several circuit parameters, such as internal fraction factor *f* or ratio of number of new input terminals to total number of new terminals $\alpha$
- More accurate total number of nets and average net degree
- Underestimate the distribution for high net degrees

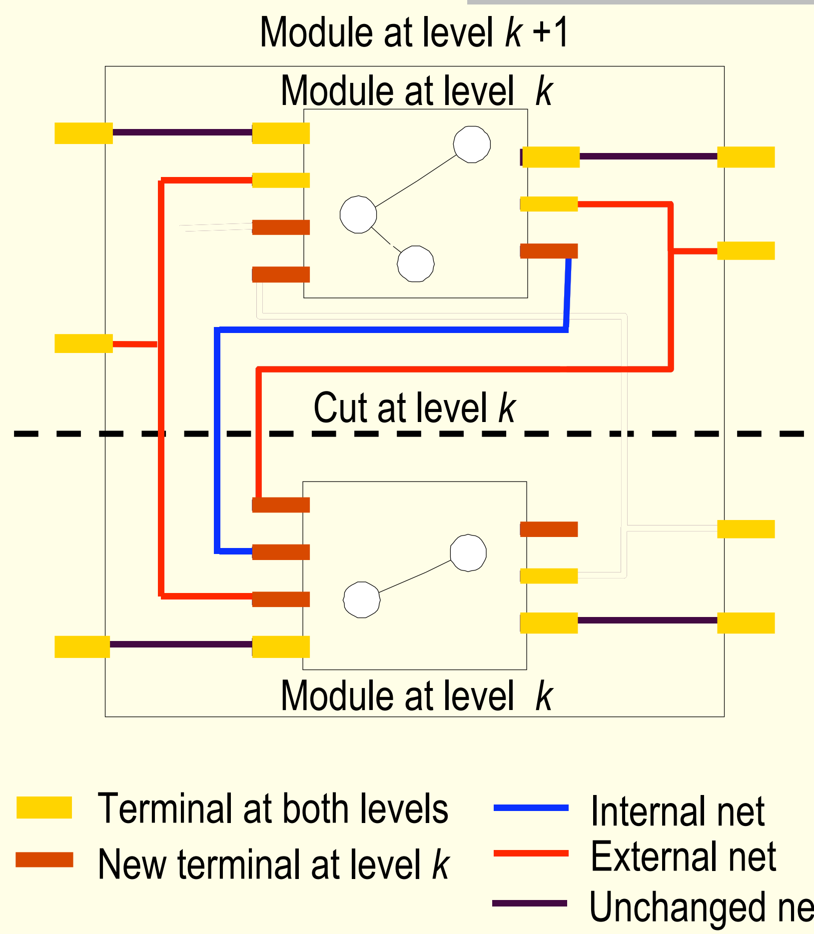
  - Maximum fan-out (maximum net degree -1)

# Stroobandt's Model

- Stroobandt's derivation is based on the relationship between the number of new terminals and the number of nets cut
- His derivation did not consider the effect of Region II of Rent's rule and did not take into account

  - technology constraint

  - psychological constraint



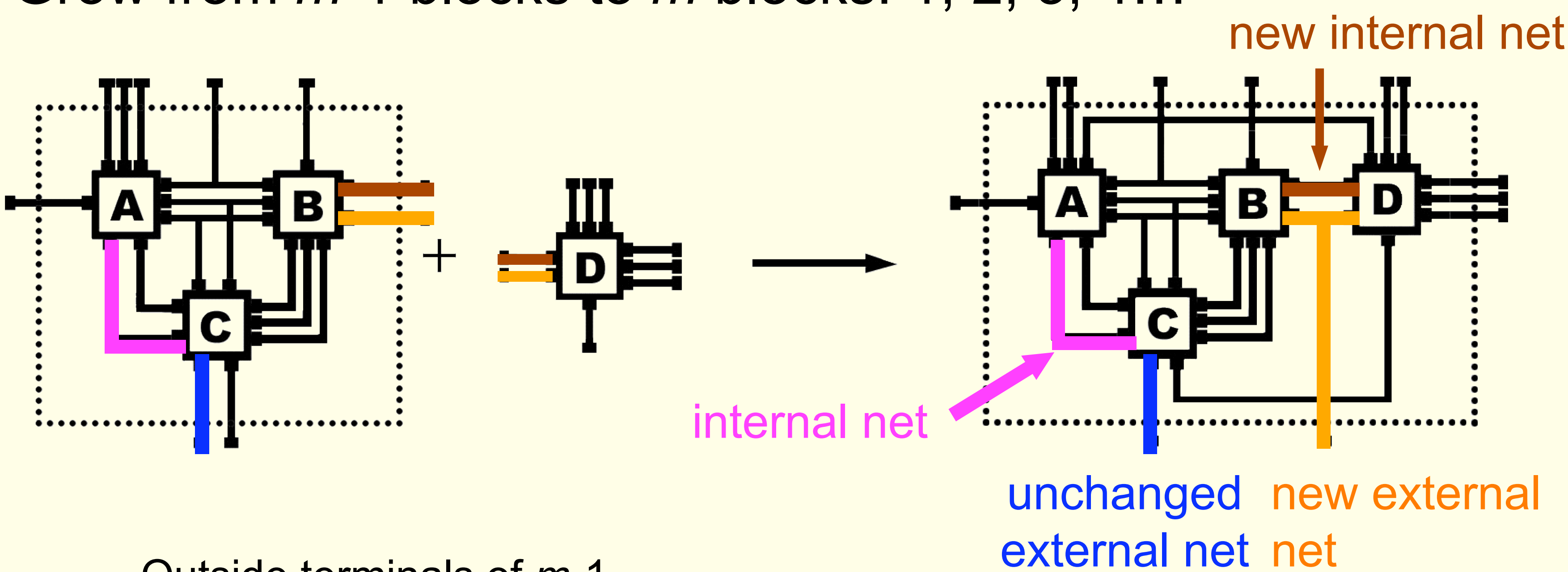

(Source: Stroobandt's paper)

# Our Models

- Zarkesh-Ha et al. were interested in the number of terminals shared through an *i*-terminal net for each block. Stroobandt analyzed the number of new terminals generated by cutting nets.
- We directly look into the net-degree distribution
  - net-degree distribution $N(i)\big|_m$
  - internal net-degree distribution $N_i(i)\big|_m$
  - external net-degree distribution $N_e(i)\big|_m$
  - their normalized expressions $nN(i)\big|_m$ , $nN_i(i)\big|_m$ and $nN_e(i)\big|_m$
- We record the change of $nN_i(i)\big|_m$ and $nN_e(i)\big|_m$ when circuit grows

# Model 1: Linear

Grow from *m*-1 blocks to *m* blocks: 1, 2, 3, 4…




Outside terminals of *m*-1 blocks have equal probability to be selected for combining

internal fraction factor *f*

$$f = \frac{\text{new internal net}}{\text{new internal} + \text{new external}}$$

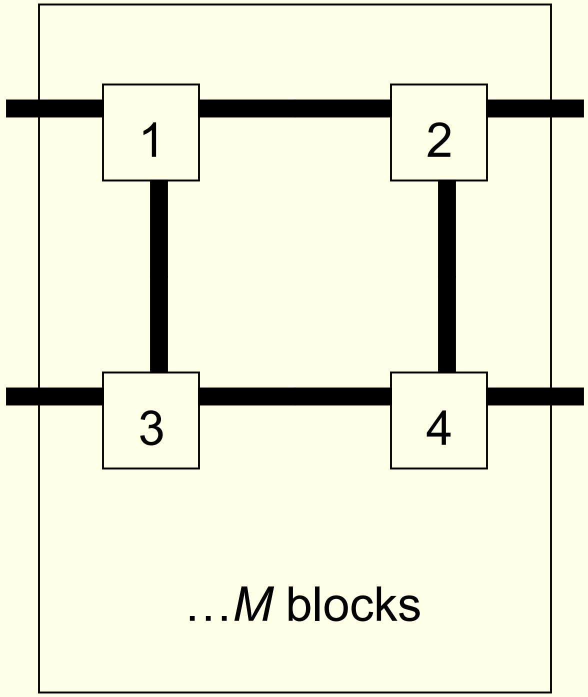
# Model 1: Linear

**Algorithm** net-degree-distribution($M$)

**Input:** $M, f, p, k$
**Output:** net-degree distribution $N$

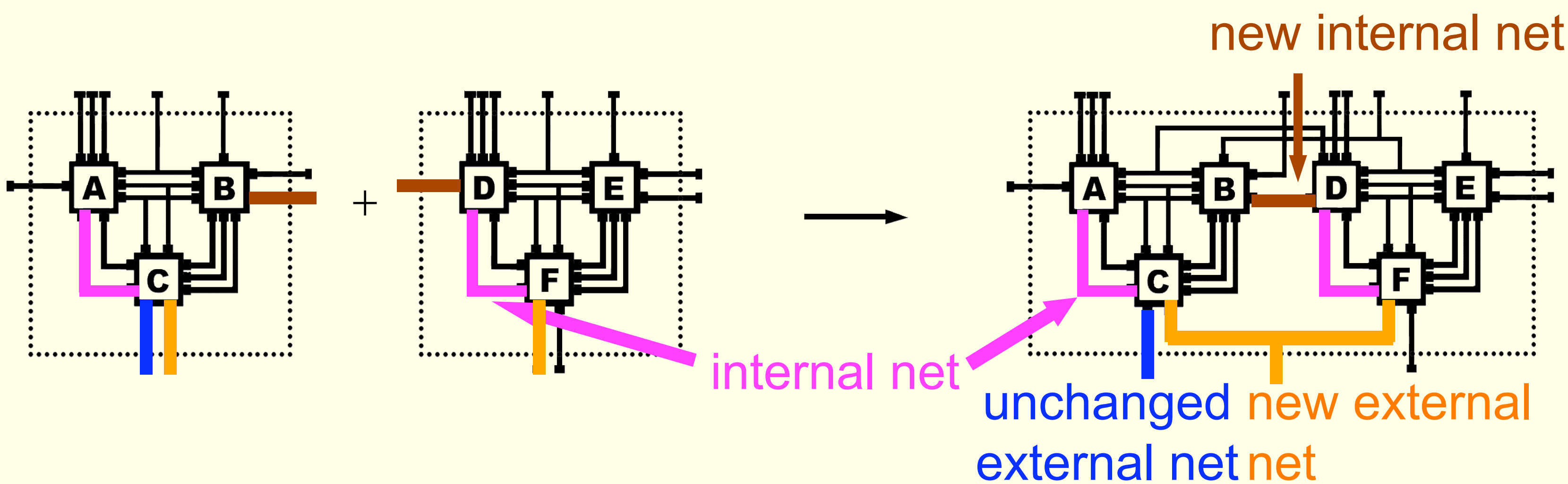
**begin**
initialize vectors: $nN_i$, $nN_e$, and $N$;
$nN_e(2)=k;$(*$nN_e$ distribution for a single block*)
**for** $i$=2 to $M$ **do**
  update $nN_i$ and $nN_e$;
**end for**
**for** $i$=2 to $M$ **do**
  $N(i)=round(nN_i(i)*M+nN_e(i)*M)$;
**end for**
**end.**

A circuit of *M* blocks



# Model 2: Exponential

Grow from *m* blocks to 2*m* blocks: 1, 2, 4, 8…




Outside terminals of *m* blocks have equal probability to be selected for combining

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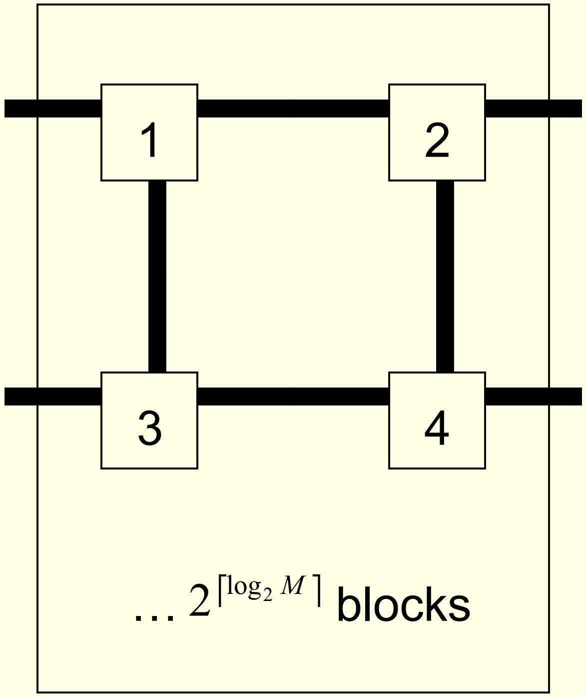
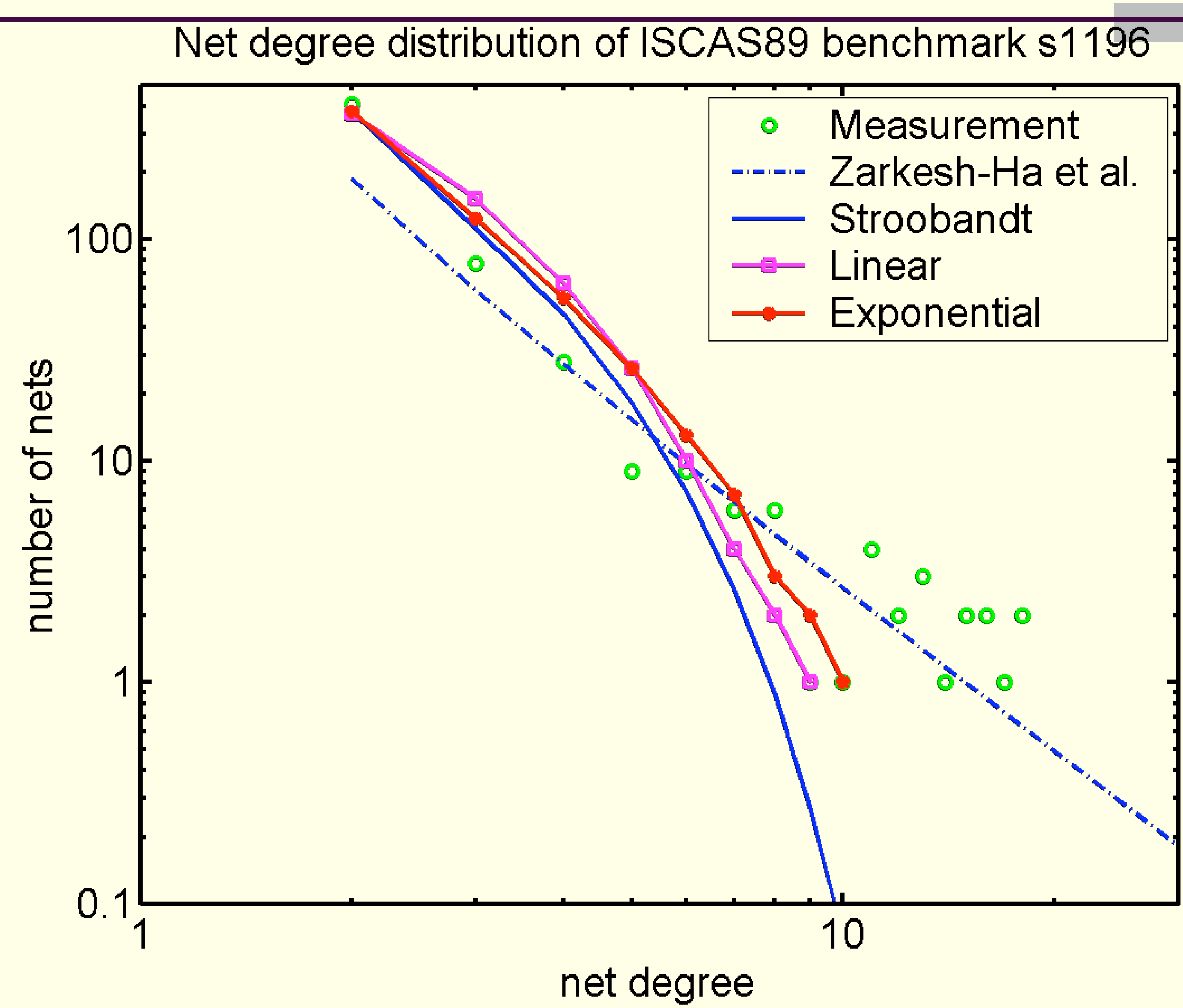
# Model 2: Exponential

| **Algorithm** net-degree-distribution($M$) |
|---|
| **Input:** $M, f, p, k$<br>**Output:** net-degree distribution $N$ |
| **begin**<br>initialize vectors: $nN_i$, $nN_e$, and $N$;<br>$nN_e(2)=k$;(*$nN_e$ distribution for a single block*)<br>$N=ceil(log_2M)$;(*Determine the number of iterations*)<br>**for** $h$=0 to $N$-1 **do**<br>    $m1=2^h$;<br>    $m2=2^{h+1}$;<br>    update $nN_i$ and $nN_e$;<br>**end for**<br>**for** $i$=2 to $M$ **do**<br>    $N(i)=round(nN_i(i)*M+nN_e(i)*M)$;<br>**end for**<br>**end.** |

A circuit of $M$ blocks



# Model 1 and Model 2

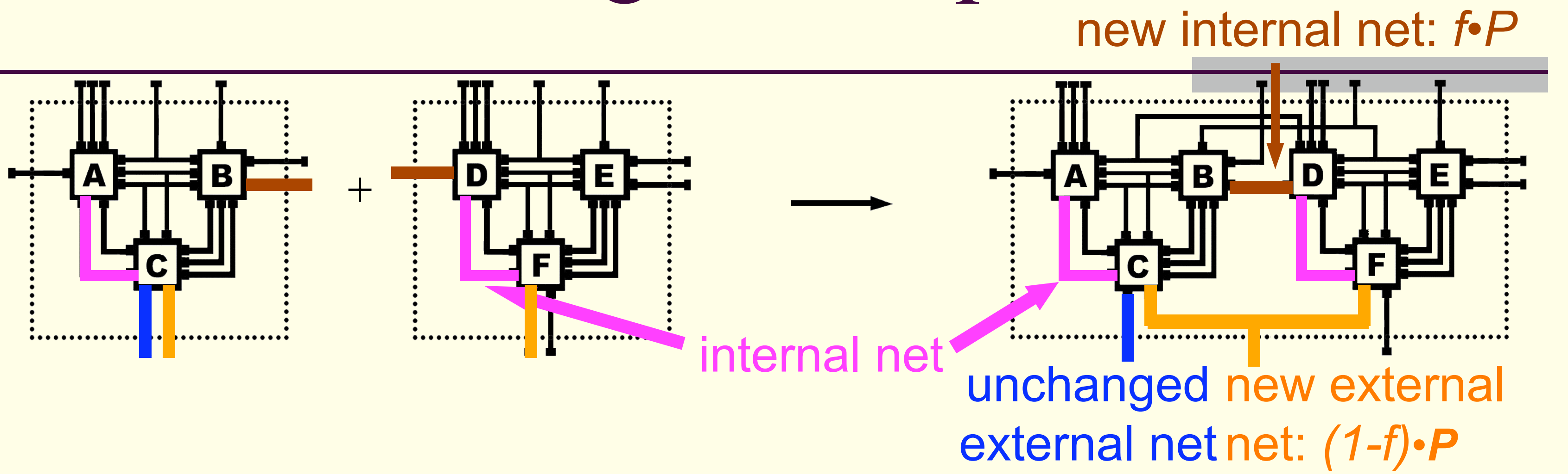


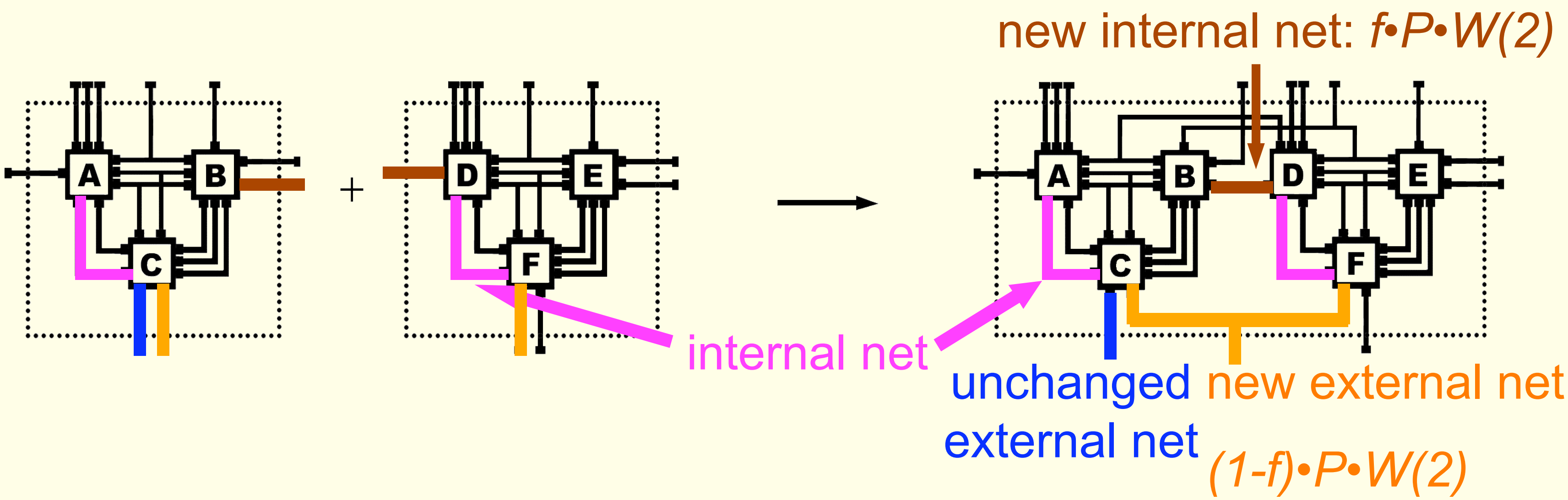
# Model 3: Weighted Exponential

- Technology constraint and psychological constraint will result in the fact that nets with higher degrees are generated with higher probability
- We defined a vector *W*(*n*) to associate the probability to the degree of newly generated nets

$$W(n) = |n - 3.5|^{0.7}$$

# Model 3: Weighted Exponential



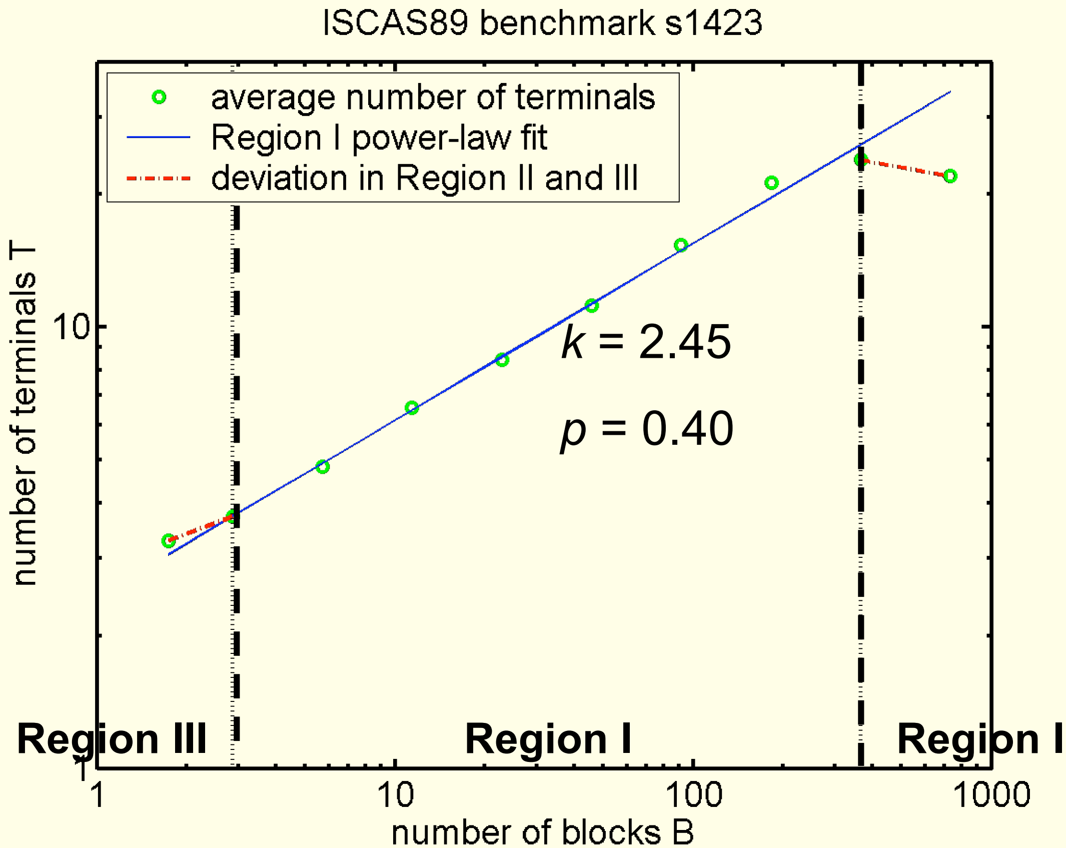



# Experimental Results

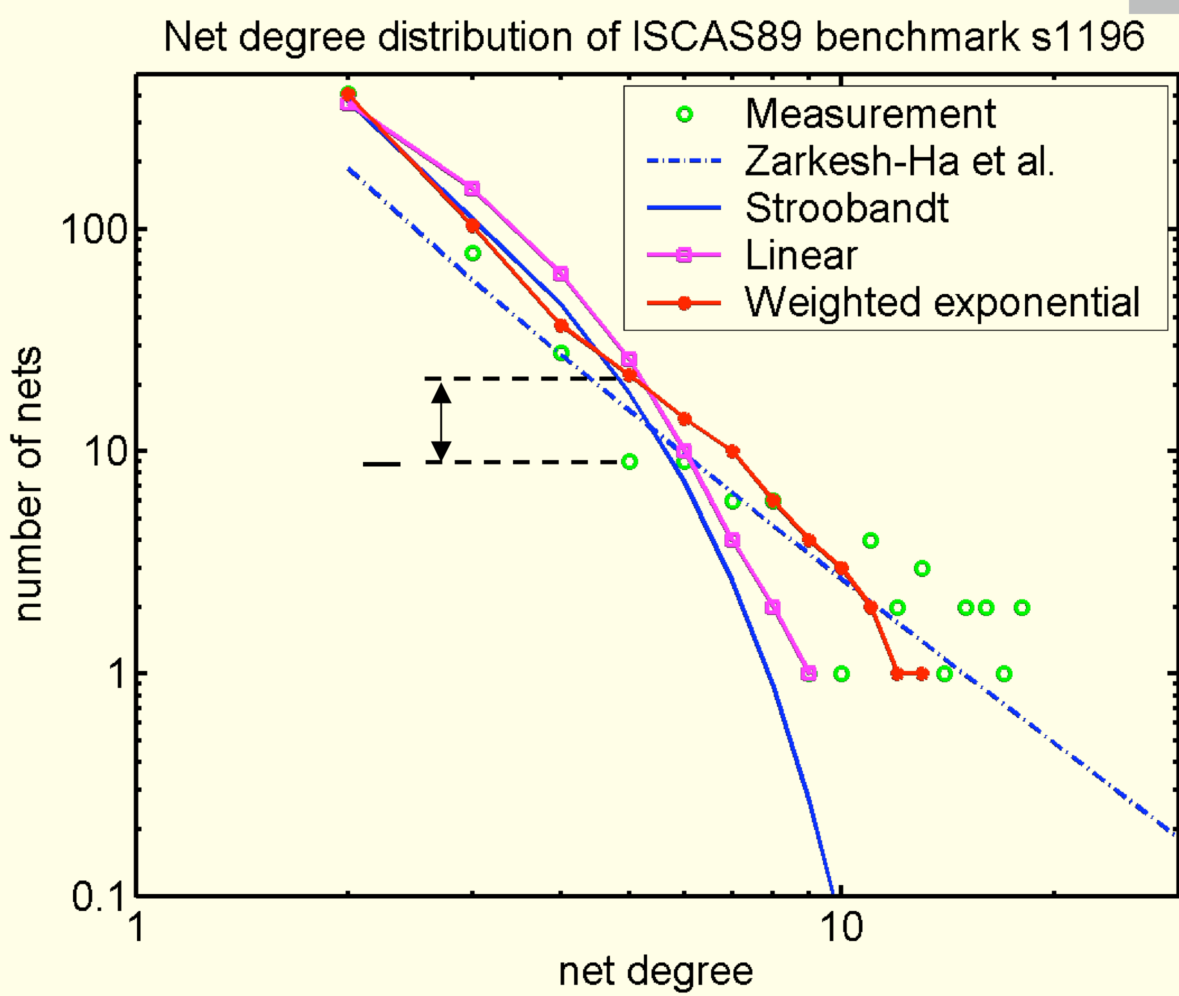
- *M*: number of circuit blocks
  *T*: number of terminals
  $N_{tot}$: total number of nets
  - found for ISCAS89 benchmarks
- *k* and *p*: Rent's parameters
  - partitioning and Region I power-law fitting
- *f*: internal fraction factor

$$f = \frac{N_{tot} - T}{kM - N_{tot}}$$

- Net-degree distribution
  - counted by a Matlab program for ISCAS89 benchmarks



# Experimental Results



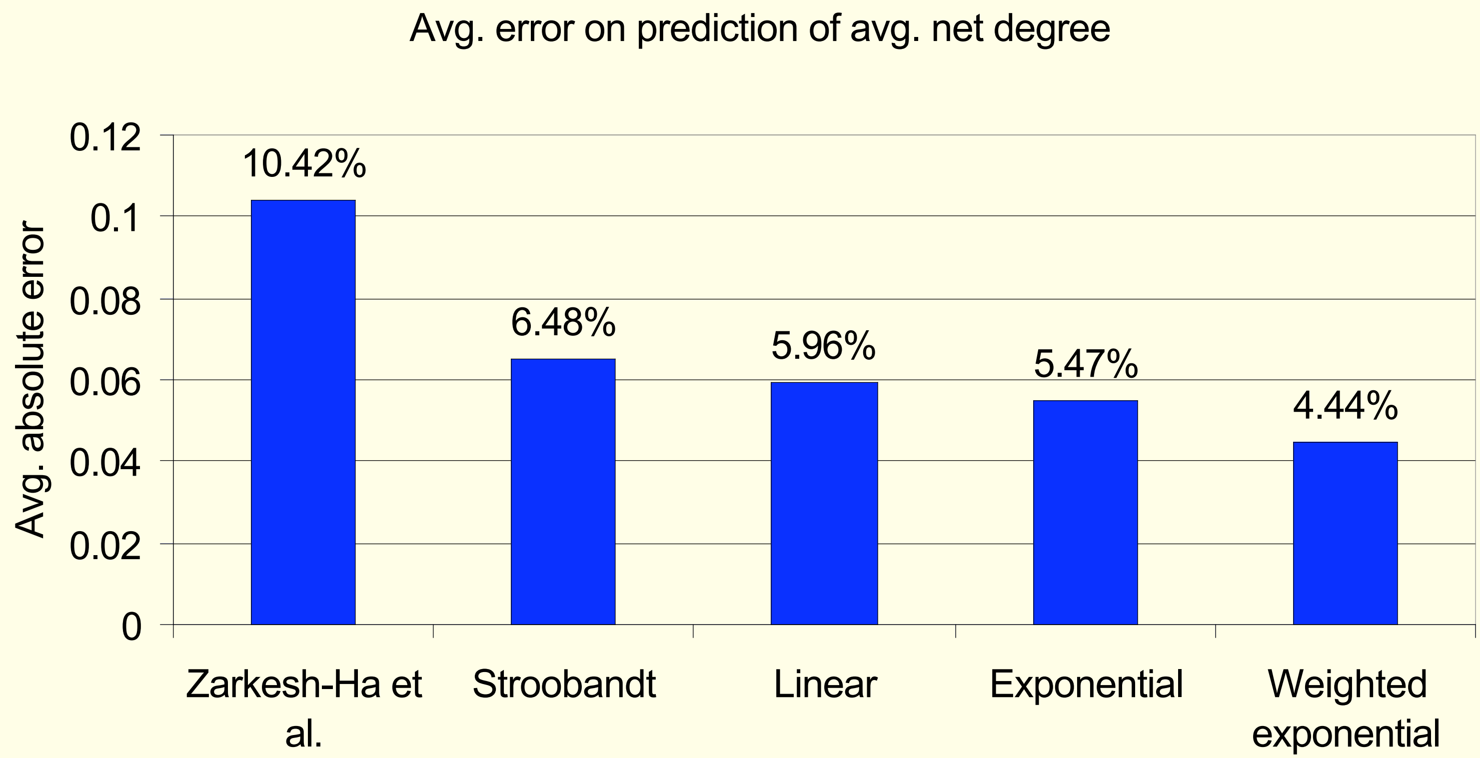
# The Least Squares Method: $\sum(\Delta^2)$

| Name | Zarkesh-Ha et al. | Stroobandt | Linear | Exponential | Weighted exponential |
|---|---|---|---|---|---|
| s27 | 37 | 0 | 0 | 0 | 0 |
| s208.1 | 1776 | 119 | 346 | 181 | 60 |
| s298 | 1781 | 567 | 1191 | 605 | 302 |
| s386 | 6195 | 3393 | 4401 | 3207 | 2126 |
| s344 | 7126 | 525 | 940 | 640 | 264 |
| s349 | 6961 | 551 | 994 | 659 | 280 |
| s382 | 2107 | 467 | 1909 | 721 | 263 |
| s444 | 2481 | 147 | 601 | 143 | 133 |
| s526 | 3512 | 1998 | 5265 | 2586 | 1345 |
| s526n | 3930 | 2051 | 5162 | 2602 | 1318 |
| s510 | 9754 | 1436 | 1112 | 842 | 1103 |
| s420.1 | 5915 | 929 | 3461 | 1431 | 555 |
| s713 | 46509 | 638 | 1202 | 710 | 162 |
| s953 | 21030 | 289 | 1783 | 901 | 1214 |

The distribution with smallest $\sum(\Delta^2)$ is the best approximation

# Average Net Degree

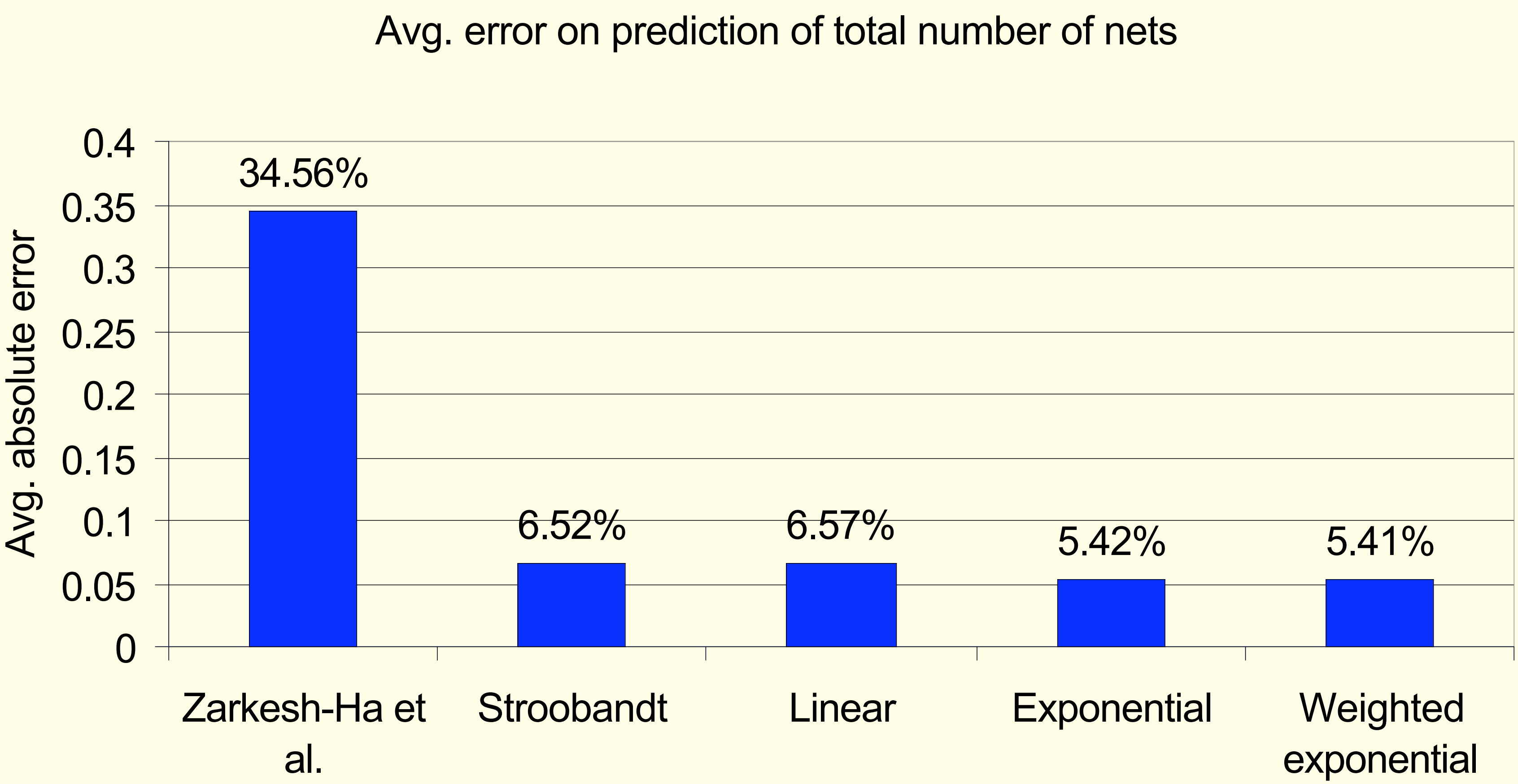
$$\text{error} = \frac{|\text{prediction-measurement}|}{\text{measurement}}$$




Average net degree and total number of nets are the most important parameters to represent interconnect complexity

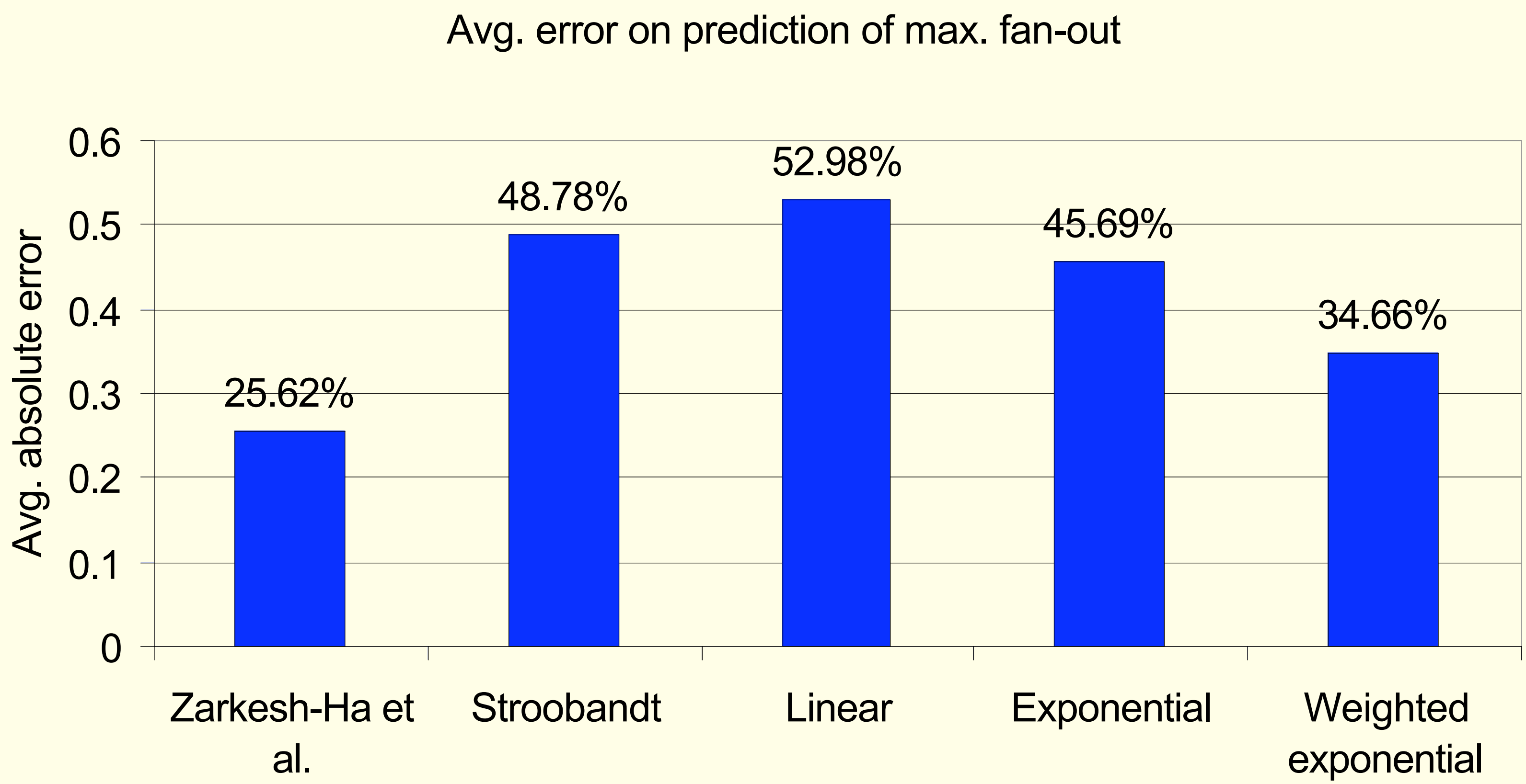
# Total Number of Nets

$$\text{error} = \frac{|\text{prediction-measurement}|}{\text{measurement}}$$



# Maximum Fan-out

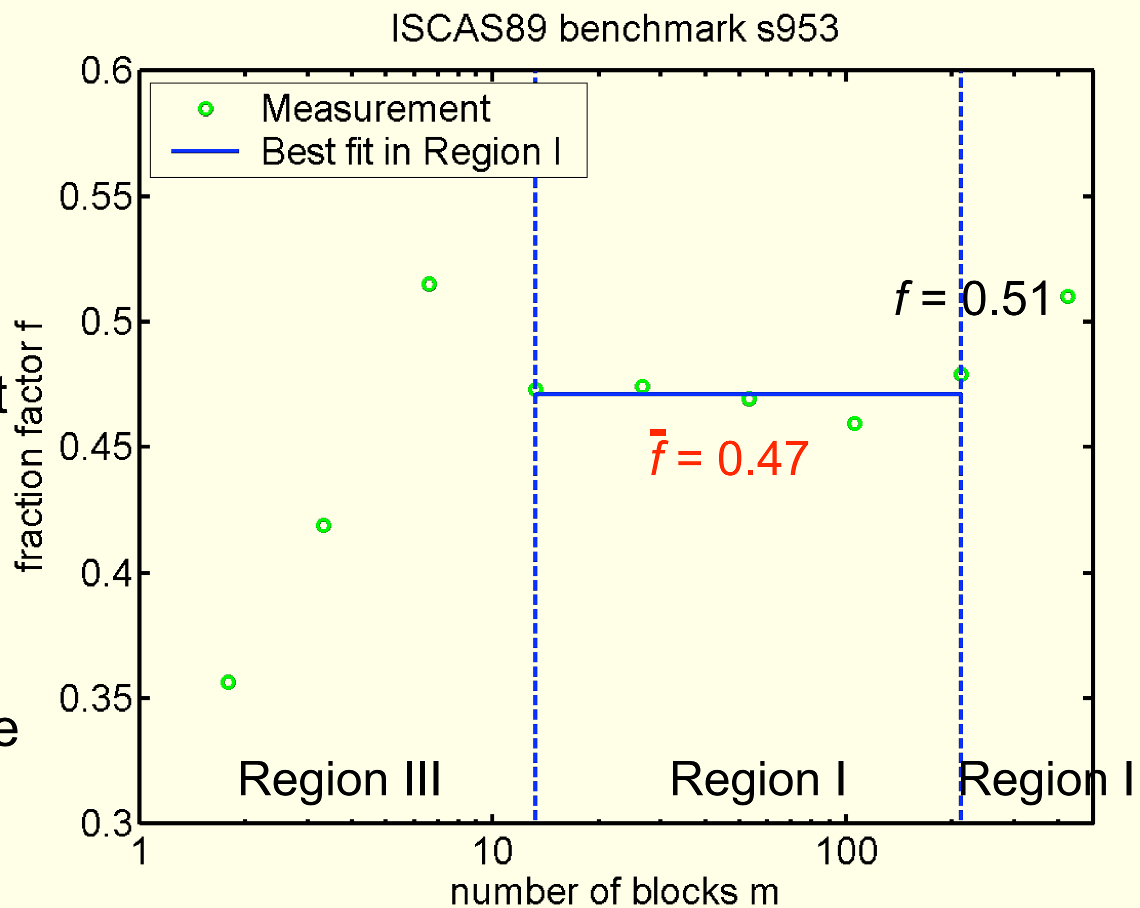
$$\text{error} = \frac{|\text{prediction-measurement}|}{\text{measurement}}$$




Maximum fan-out is important to predict the length of the longest path

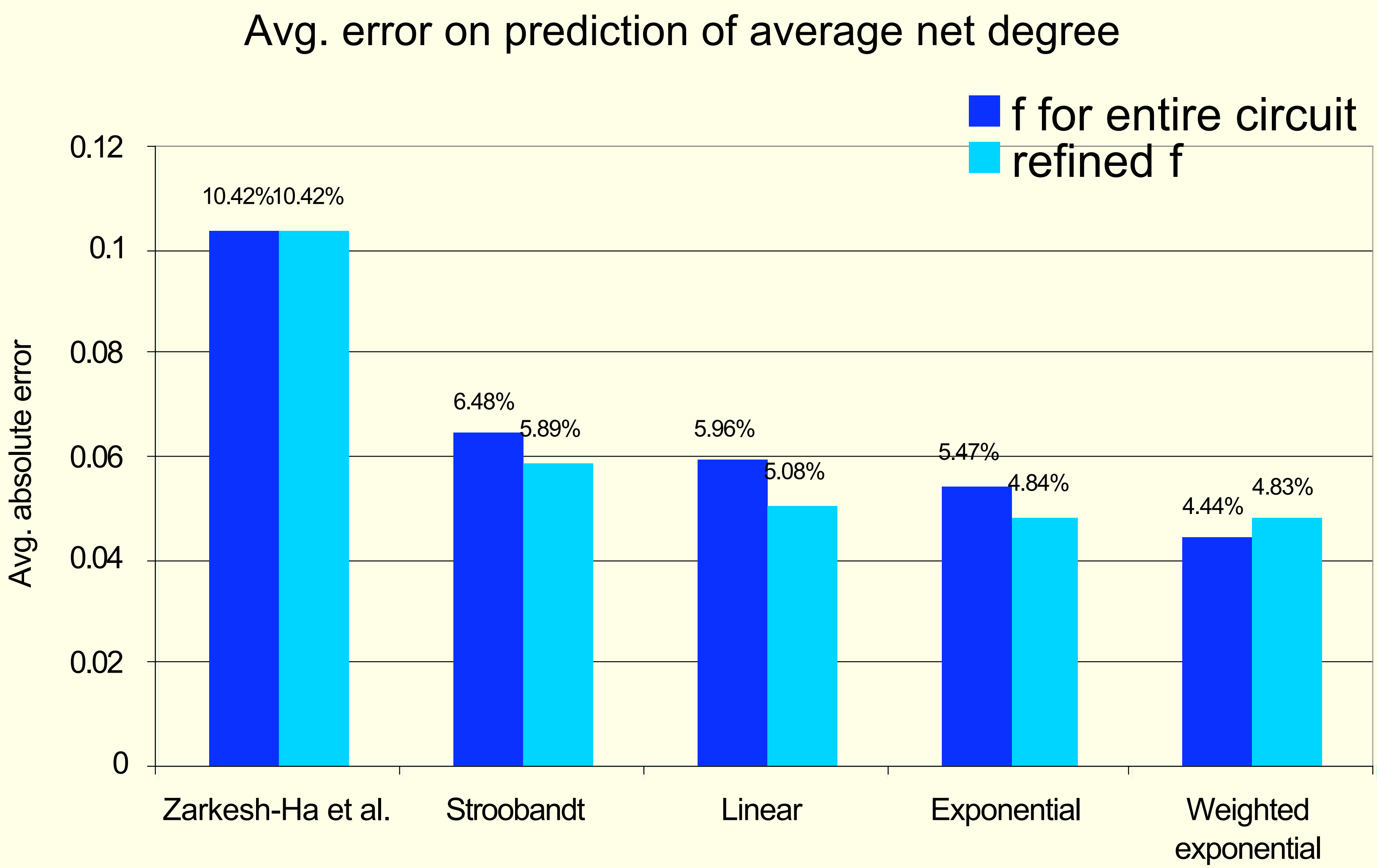
# Behavior of Internal Fraction Factor *f*

- Internal fraction factor *f,* similarly to Rent's rule, exhibits Region II and Region III
- We applied a best fitting procedure to get a refined value of *f.*
- With the refined value of *f,* our models and Stroobandt's model can provide even more accurate estimation

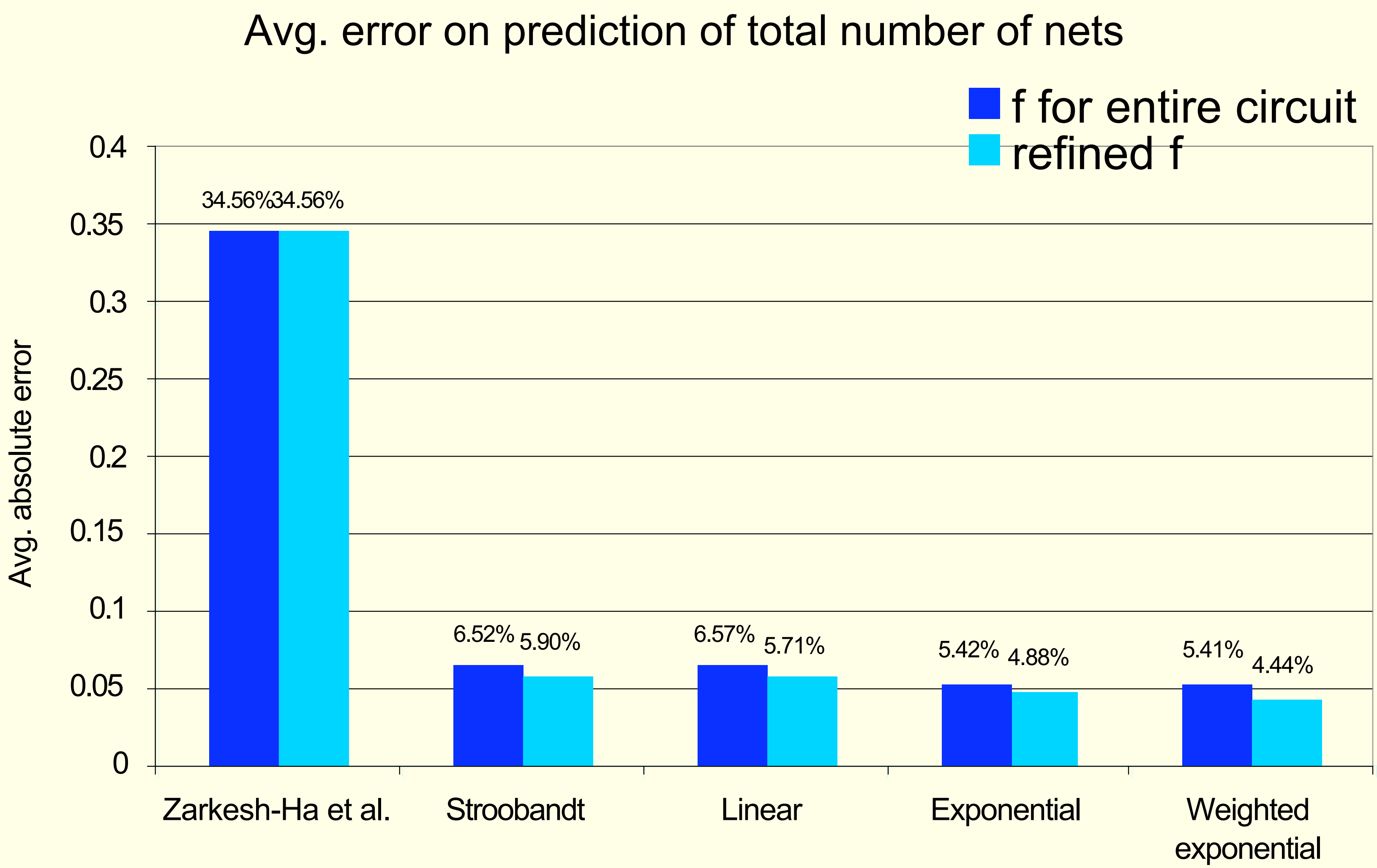



$$f = \frac{\text{new internal net}}{\text{new internal + new external}}$$

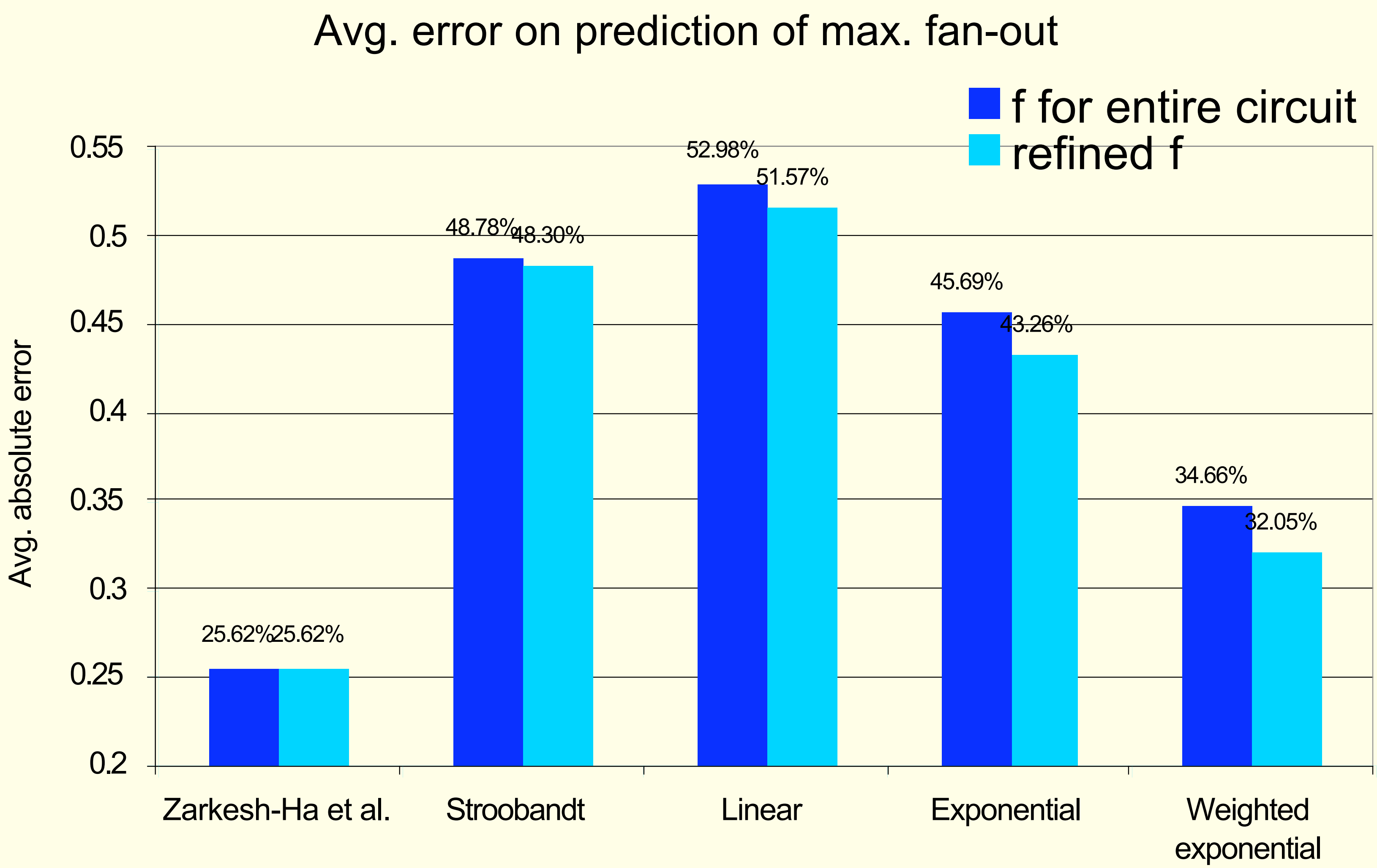
# Comparison on Average Net Degree



# Comparison on Total Number of Nets



# Comparison on Maximum Fan-out



# Conclusions

- We found an alternative way to derive net-degree distribution models by directly using normalized net-degree distribution
  - linear growth model
  - exponential growth model
  - weighted exponential model
- We accurately measured Rent's parameters *k* and *p*, internal fraction factor *f* and net-degree distribution of ISCAS89 benchmarks, which can be good references for other researchers

# Conclusions

- For the first time, we observed that internal fraction factor *f*, similarly to Rent's rule, also exhibited Region II and Region III. With a refined value of *f*, our models and Stroobandt's model can provide even more accurate estimation of net-degree distribution

- Weighted exponential growing model is more accurate than any other currently available net-degree distribution model

| | Avg. net degree | Total num. of nets | Max. fan-out |
|---|---|---|---|
| Zarkesh-Ha et al. | 56.65% | 87.15% | -25.10% |
| Stroobandt | 18.00% | 24.75% | 28.95% |

Accuracy enhancement of weighted exponential model

$$\text{accuracy enhancement} = \frac{\text{avg. error of Str.} - \text{avg. error of weighted}}{\text{avg. error of Stroobandt}}$$

# Conclusions

- Weighted exponential model is a general theoretical model

  - applicable to various levels of circuit descriptions and various physical architectures

    - system level, circuit level, physical level

    - copper interconnect, optical interconnect

  - lots of possible applications

    - one example: a benchmark generator for floorplanning. Our tool, Bgen, generates more realistic benchmark circuits using weighted exponential model than when using other models