Maximum Multiplicity Distribution (MMD)

Pranav Anbalagan
Dr. Jeff Davis

Georgia Institute of Technology

The authors gratefully acknowledge the support of the National Science Foundation (NSF# 0098227) for this research.
Rentian Length Distribution

C7552 Benchmark Circuit: 3512 Gates; 5836 Interconnects

Rentian Distribution (Rents exponent 0.6)

Length Distribution

C7552 Benchmark Circuit: 3512 Gates; 5836 Interconnects

Length Frequency Histogram

- Total Length Optimized Distribution (TLOD)
- Rentian Distribution (Rents exponent 0.6)
Length Distribution

C7552 Benchmark Circuit: 3512 Gates; 5836 Interconnects

Total Length Optimized Distribution (TLOD)
Rentian Distribution (RD)
MMD Real (RMMD)
Salient Properties of New Distribution

- Reduction in maximum length of interconnects
- Reduction in number of long interconnects
- Increases the available design space
Outline

- Multiplicity
- Theoretical MMD
- Real MMD
- Impact on Global Interconnects
- Conclusion
Multiplicity of Length Distribution

Total number of layout arrangements that have the same length distribution
Interconnect Positions

- **M[L]**: Number of interconnect positions of length L in a layout
  - **M[1]** = 4
  - **M[2]** = 2
Interconnect Positions

- **Density of interconnect states function** $M[L]$

  $1 \leq L < \sqrt{N}$:
  
  $$M[L] = \frac{L^3}{3} - 2L^2 \sqrt{N} + \frac{1}{3} L(6N - 1)$$

  $\sqrt{N} \leq L < 2\sqrt{N} - 2$:
  
  $$M[L] = \frac{L^3}{3} + 2L^2 \sqrt{N} - \frac{1}{3} L(12N - 1) + \frac{2}{3} \sqrt{N}(4N - 1)$$

---

Multiplicity of Length Distribution

- **Assumptions**
  - Gates are arranged in a *homogeneous square layout array*
  - Interconnects are *independent*
  - Each interconnect is *distinct*
Independent Interconnects

- $N[1] = 2$

- Impossibilities Counted (Simple Calculations)!
Distinct Interconnects

- \( N[1] = 2 \)

- *Unique* arrangement should be counted!
## Multiplicity of Length Distribution

- $N[1]=2$
  
- $M[1]=4$
  
- $4 \times 3 \ \{M[1] \times (M[1]-1)\}$ ways of arranging the 2 interconnects

\[
\Omega_L = \prod_{I=0}^{N[L]-1} (M[L] - I)
\]

- Multiplicity of interconnects of length $L$

\[
\Omega_L = \frac{M[L]!}{(M[L] - N[L])!}
\]
Multiplicity Calculation

- **Multiplicity of interconnect length distribution**
  \[
  \Omega = \prod_{L=1}^{2\sqrt{N} - 2} \frac{M[L]!}{(M[L] - N[L])!}
  \]

- **Entropy**
  \[
  S = \sum_{L=1}^{L_{\text{MAX}}} \log(M[L]!) - \log((M[L] - N[L])!)
  \]
C7552 Benchmark Circuit: 3512 Gates; 5836 Interconnects

Multiplicity of \( RD = 10^{20062} \) < Multiplicity of \( TLOD = 10^{21858} \) <

Multiplicity of \( RMMD = 10^{23084} \)
Outline

- Multiplicity
- *Theoretical MMD*
- Real MMD
- Impact on Global Interconnects
- Conclusion
MMD Problem Definition

Find the values of $N(L)$ such that $S$

$$S = \sum_{L=1}^{L_{\text{MAX}}} \log(M[L]!) - \log((M[L] - N[L])!)$$

is maximized under the following constraints

$$N_{\text{Edges}} = \sum_{L=1}^{L_{\text{MAX}}} N[L] \quad L_{\text{Total}} = \sum_{L=1}^{L_{\text{MAX}}} L \times N[L]$$
MMD Derivation

\[ f_1 = \sum_{L=1}^{L_{\text{MAX}}} N[L] - N_{\text{Edges}} \]  
\[ f_2 = \sum_{L=1}^{L_{\text{MAX}}} L \cdot N[L] - L_{\text{Total}} \]

\[ \frac{\partial S}{\partial N[L]} + \alpha \frac{\partial f_1}{\partial N[L]} + \beta \frac{\partial f_2}{\partial N[L]} = 0 \]  
\[ \log(K!) = K \log(K) - K \]  
\[ \frac{\partial \log((M[L] - N[L])!)}{\partial N[L]} = -\log(M[L] - N[L]) \]  

\[ \frac{\partial \log((M[L] - N[L])!)}{\partial N[L]} + \alpha + \beta L = 0 \]  
\[ -\log(M[L] - N[L]) + \alpha + \beta L = 0 \]  
\[ N[L] = M[L] - e^\alpha + \beta L \]  
\[ \alpha = e^a \quad \beta = e^b \]
MMD Expression

\[ N[L] = M[L] - a(b)^L \]

1 ≤ L < \( \sqrt{N} \):

\[ M[L] = \frac{L^3}{3} - 2L^2 \sqrt{N} + \frac{1}{3} L(6N - 1) \]

\( \sqrt{N} \leq L < 2\sqrt{N} - 2 \):

\[ M[L] = \frac{L^3}{3} + 2L^2 \sqrt{N} - \frac{1}{3} L(12N - 1) + \frac{2}{3} \sqrt{N} (4N - 1) \]
MMD Calculation

\[ N[L] = \text{Max}(M[L] - a(b)^L, 0) \]

\[ N_{\text{Edges}} = \sum_{L=1}^{L_{\text{MAX}}} \text{Max}(M[L] - a(b)^L, 0) \]

\[ L_{\text{Total}} = \sum_{L=1}^{L_{\text{MAX}}} L \times \text{Max}(M[L] - a(b)^L, 0) \]
MMD Calculation

MMD evolution for c7552 circuit with decreasing average length

TLOD and MMD for c7552 benchmark circuit with average length of 4.89
Outline

- Multiplicity
- Theoretical MMD
- *Real MMD*
- Impact on Global Interconnects
- Conclusion
Real MMD

- Distribution Driven Placement

Cost = \sum W \times |N_{act}[L] - N_{req}[L]|

<table>
<thead>
<tr>
<th>W values</th>
<th>Lact &lt; Lmin</th>
<th>Lmin &lt;= Lact &lt;= Lmax</th>
<th>Lmax &lt; Lact</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cost 1</strong></td>
<td>Lmin-Lact</td>
<td>Lact</td>
<td>Lact-Lmax</td>
</tr>
<tr>
<td><strong>Cost 2</strong></td>
<td>Lmin-Lact</td>
<td>1</td>
<td>Lact-Lmax</td>
</tr>
<tr>
<td><strong>Cost 3</strong></td>
<td>Lmin-Lact</td>
<td>0</td>
<td>Lact-Lmax</td>
</tr>
<tr>
<td><strong>Cost 4</strong></td>
<td>Lact</td>
<td>Lact</td>
<td>Lact</td>
</tr>
</tbody>
</table>
Distribution Driven Placement

TLOD, Theoretical MMD, MMD-Distribution Matched (Real MMD) using cost function 2 and cost function 3

<table>
<thead>
<tr>
<th>Circuit Name: C499</th>
<th>Cost1</th>
<th>Cost2</th>
<th>Cost3</th>
<th>Cost4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative % error</td>
<td>50.88</td>
<td>31.94</td>
<td>440.1</td>
<td>36.84</td>
</tr>
<tr>
<td>% Increase in total length</td>
<td>27.65</td>
<td>17.43</td>
<td>29.64</td>
<td>29.51</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>W values</th>
<th>Lact &lt; Lmin</th>
<th>Lmin &lt;= Lact &lt;= Lmax</th>
<th>Lmax &lt; Lact</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost1</td>
<td>Lmin-Lact</td>
<td>Lact</td>
<td>Lact-Lmax</td>
</tr>
<tr>
<td>Cost2</td>
<td>Lmin-Lact</td>
<td>1</td>
<td>Lact-Lmax</td>
</tr>
<tr>
<td>Cost3</td>
<td>Lmin-Lact</td>
<td>0</td>
<td>Lact-Lmax</td>
</tr>
<tr>
<td>Cost4</td>
<td>Lact</td>
<td>Lact</td>
<td>Lact</td>
</tr>
</tbody>
</table>
Outline

- Multiplicity
- Theoretical MMD
- Real MMD
- Impact on Global Interconnects
- Conclusion
Multilevel Interconnect Architecture

Global Interconnects

Local Interconnects
Global Interconnect

- Global Interconnect Cutoff = 0.2x \( L_{\text{MAX}} \)
- Global Interconnect Count - average improvement: 54\% (Cost 2) vs 68\% (Cost 3)
- Global Interconnect Length - average improvement: 55\% (Cost 2) vs 68\% (Cost 3)
Total Interconnect Length

- Average increase in total length: 21% (cost 2) vs. 23% (cost 3)
- Interconnects with what length cause an increase in total length?
- What is the least length that we could use as global interconnect cutoff and see an improvement in global interconnect?
Global Interconnect Quality

- **Global Length Cutoff (X)**
  - **Total length of interconnects with length >= X in TLOD**
  - **Total length of interconnects with length >= X in Real MMD**

- **Xg=7**

- Data points:
  - 7, 37.34
  - 7, 37.34
  - 7, 54.05
  - 24, 1086
  - 24, 3610

- Graph showing the total length of interconnects with length >= X in TLOD and Real MMD.
Global Interconnects

% Increase in total length of interconnects with length >= X

Global Length Cutoff (X)
Global Interconnect Quality

Maximum Length

Maximum interconnect length resulting from cost function 2, cost function 3 and TLOD for various circuits.
Conclusion

- New interconnect length distribution
- Reduction in
  - Number of long interconnects (55%)
  - Total length of long interconnects (55%)
  - Maximum length of long interconnects (12%)
- Increase in total interconnect length due to local interconnects
- Insight into Distribution driven Physical Design
Questions