Managing Interconnect Resources

Embedded SLIP Tutorial

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Overview

- Performance model
- Netlists and signatures
- Partitioning and placement
- Rent exponents
- What do you want to model today?

Performance model

RISC/CISC

$$\text{Instructions per second (M)IPS} = \frac{1}{\text{Cycle time (CT)} \times \text{Cycles per instruction (CPI)}}$$

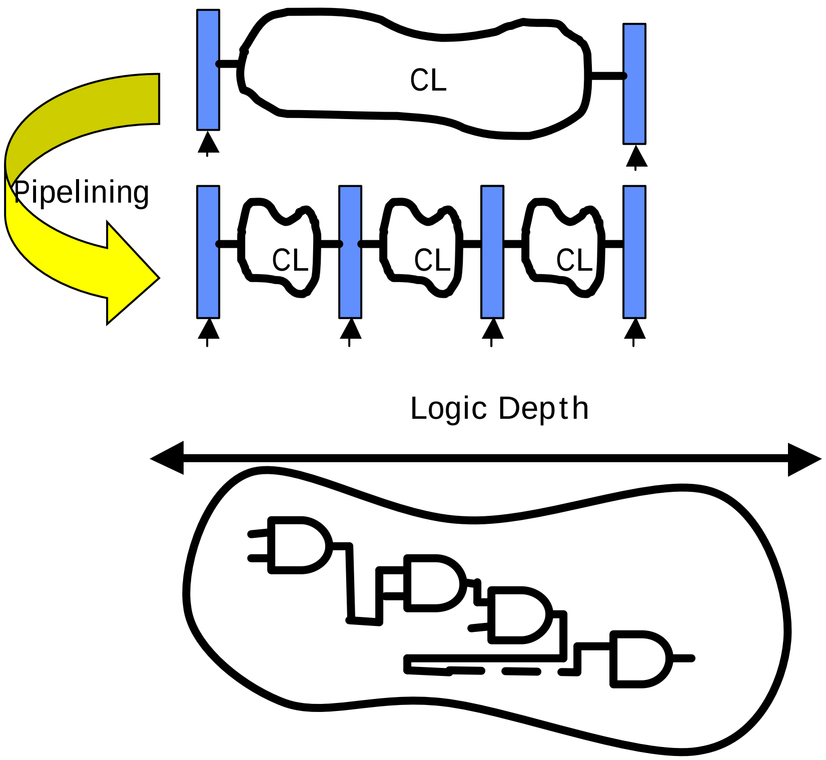
Lower CPI $\rightarrow$ more complex CPU
$\rightarrow$ internal parallelism,
branch prediction,
cache

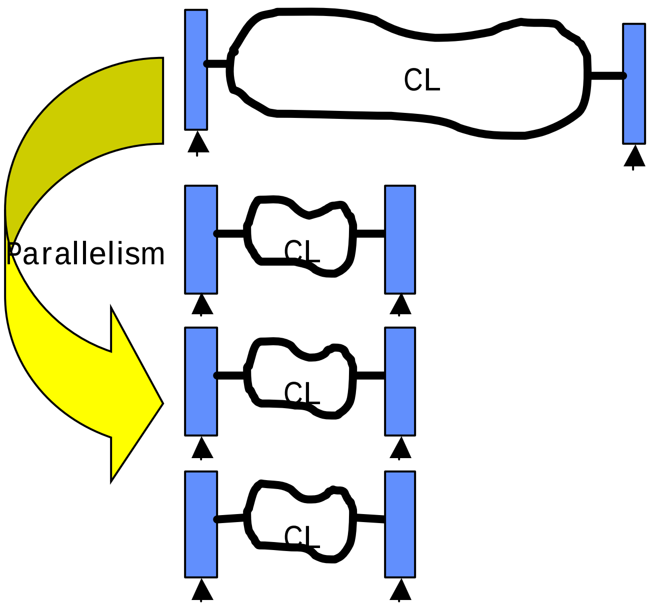
CISC CPI $\approx$ 3, RISC CPI<1

10% reduction in CPI $\rightarrow$ 20-40% increase in circuit count

Larger circuits have longer cycle times

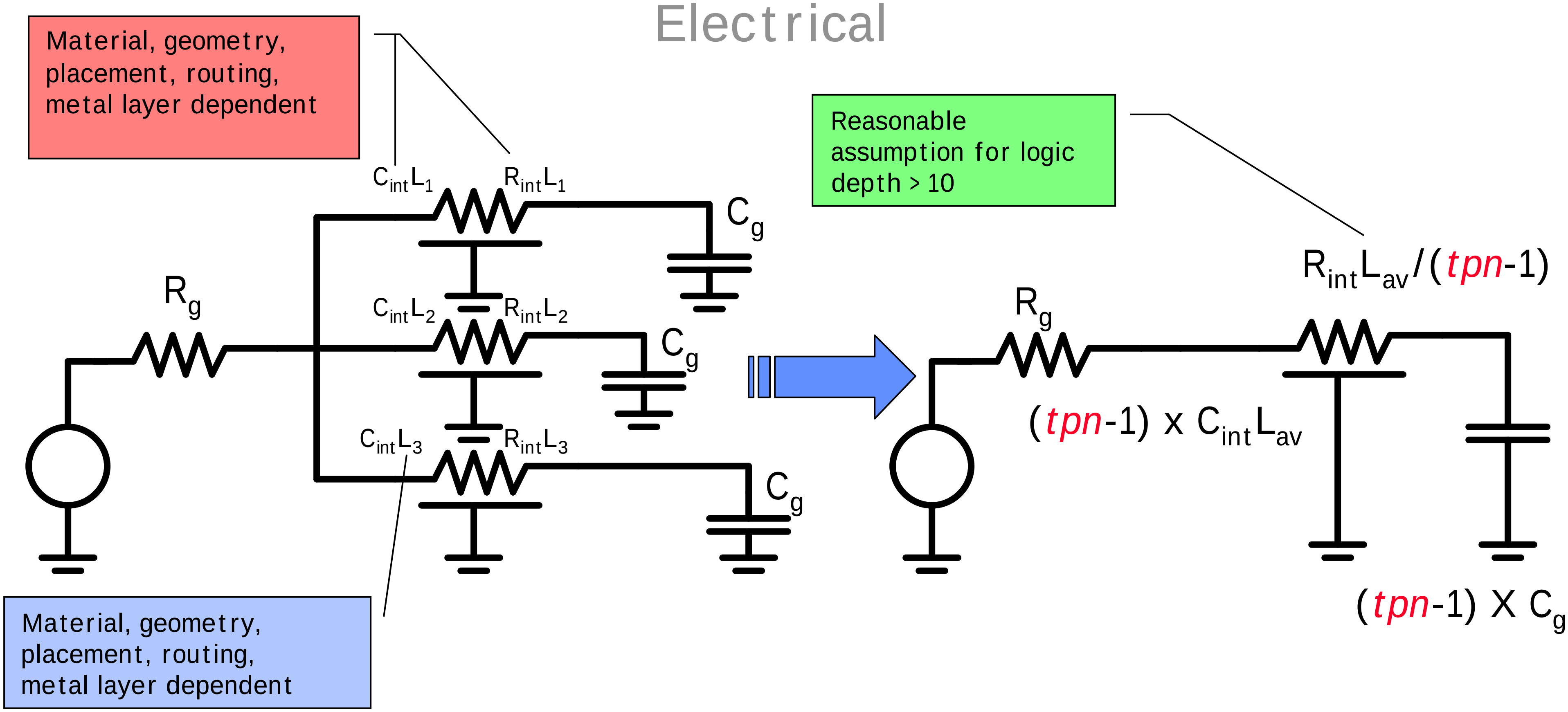
Performance model
Logic





Performance model

Electrical

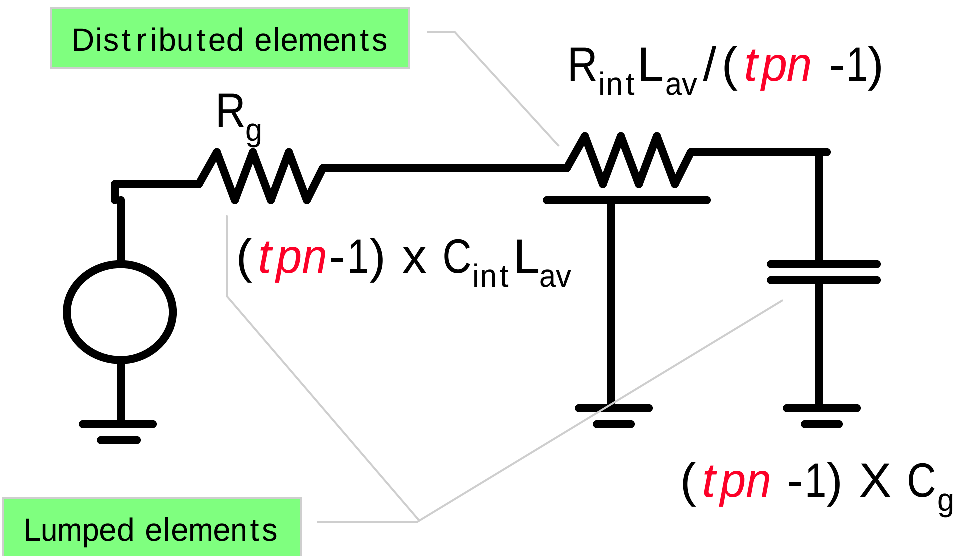


Fan-out = (terminals per net) - 1

= *tpn* - 1

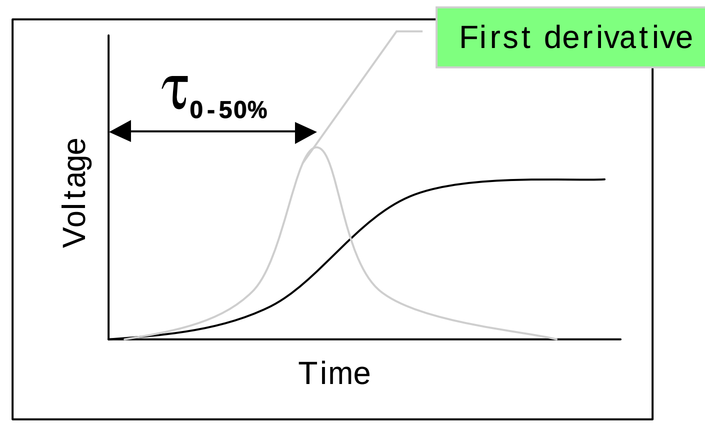
Performance model

Elmore delay



Exact solution to problem in S-domain

But no known inverse Laplace Transform back to time domain

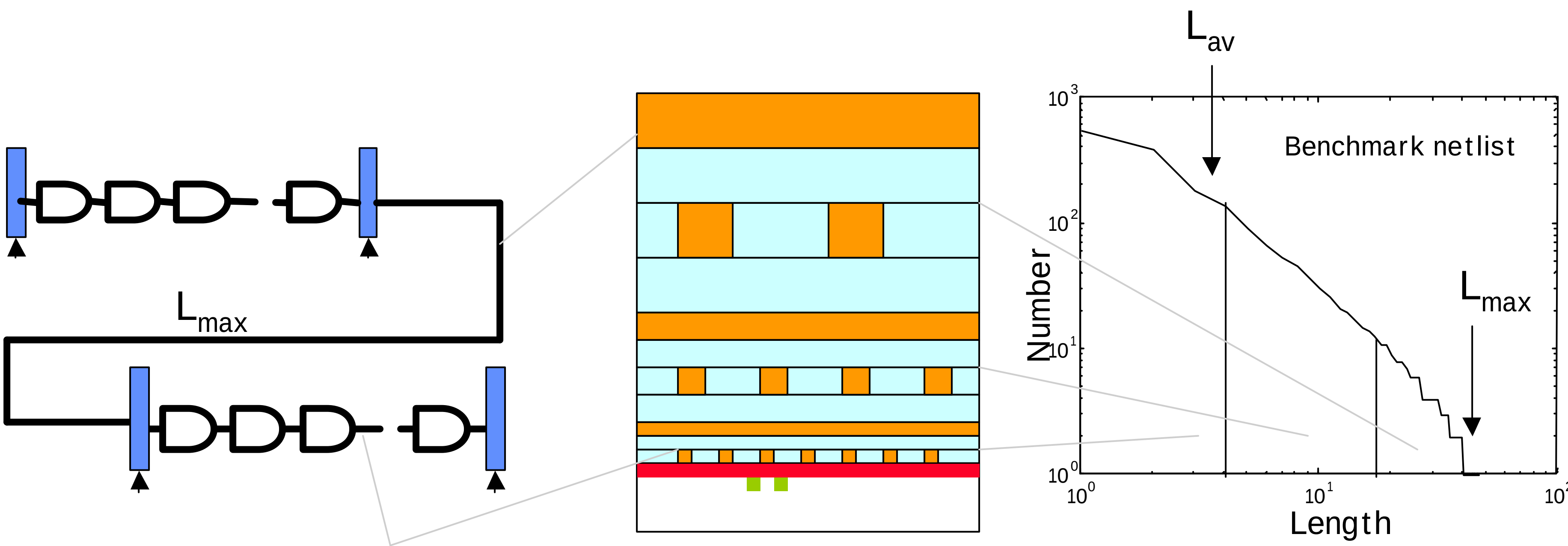


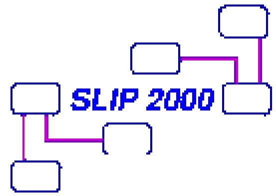
$$\tau_{0-50\%} = (tpn-1)R_gC_{int}L_{av} + (tpn-1)R_gC_g + 0.5R_{int}C_{int}L^2_{av} + R_{int}C_gL_{av}$$

Performance model

Electrical optimization

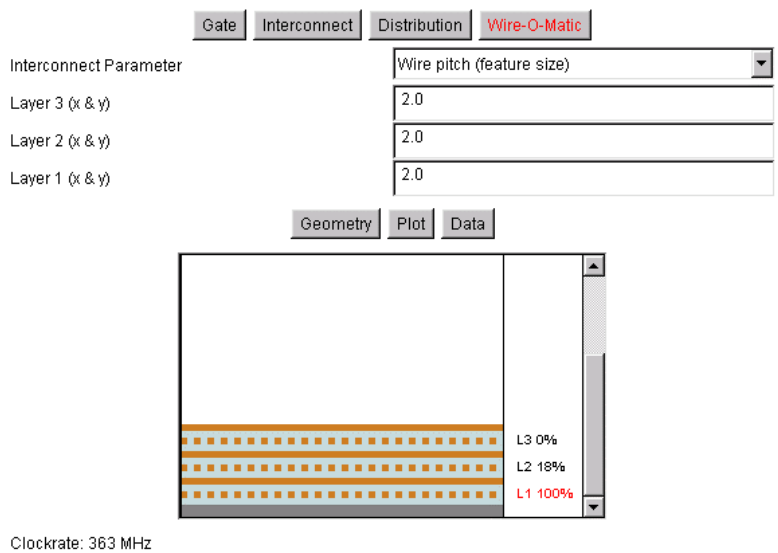
Cycle time = (logic depth) x $\tau_{0\text{-}50\%}$ + $0.5R_{int}C_{int}L^2_{max}$

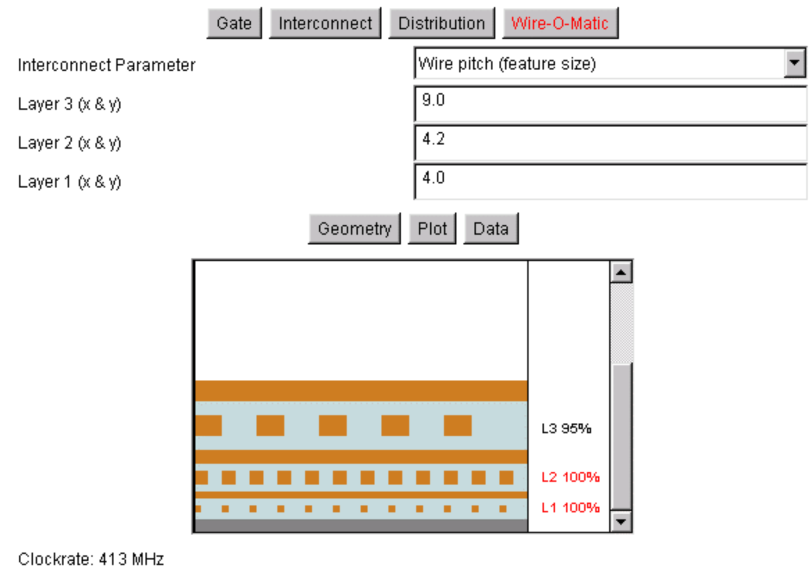




Performance model

Interconnect optimization





Basic cycle time models provide insight into the complex interactions which determine cycle time.

Modelling process can also be used to optimise power dissipation in the interconnect

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Performance model

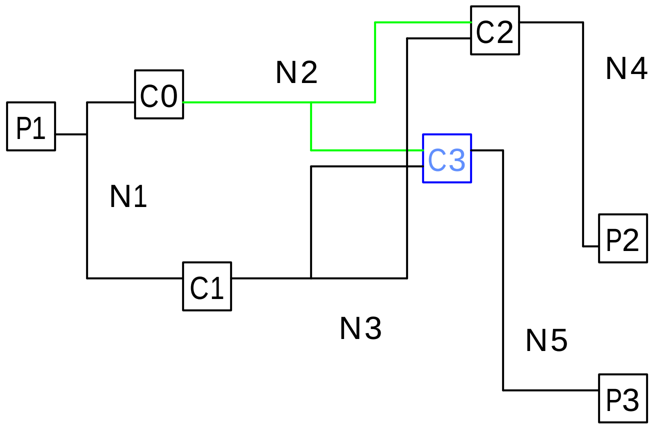
Predictive capability

- How do we know if benchmark is good?
- Is geometry optimization sensitive to netlist signature?
- What if layout tools change?
- What if we wish to analyse performance of a netlist that does not yet exist?

Netlists and signatures

formats

Net list
N1 P1 C0 C1
N2 C0 C2 C3
N3 C1 C2 C3
N4 C2 P2
N5 P3C3



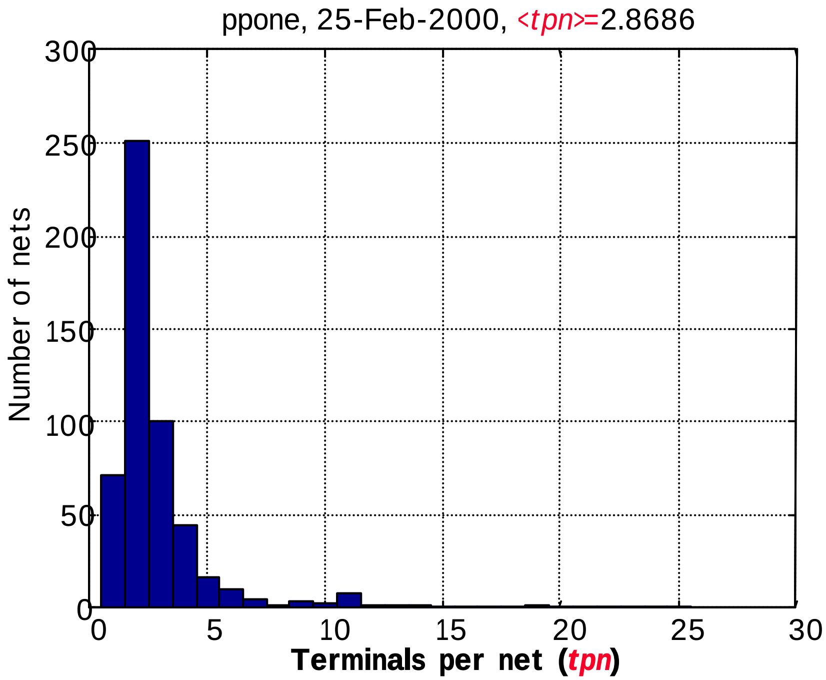
Node list
P1 N1
P2 N4
P3 N5
C0 N1 N2
C1 N1 N3
C2 N2 N3 N4
C3 N2 N3 N5

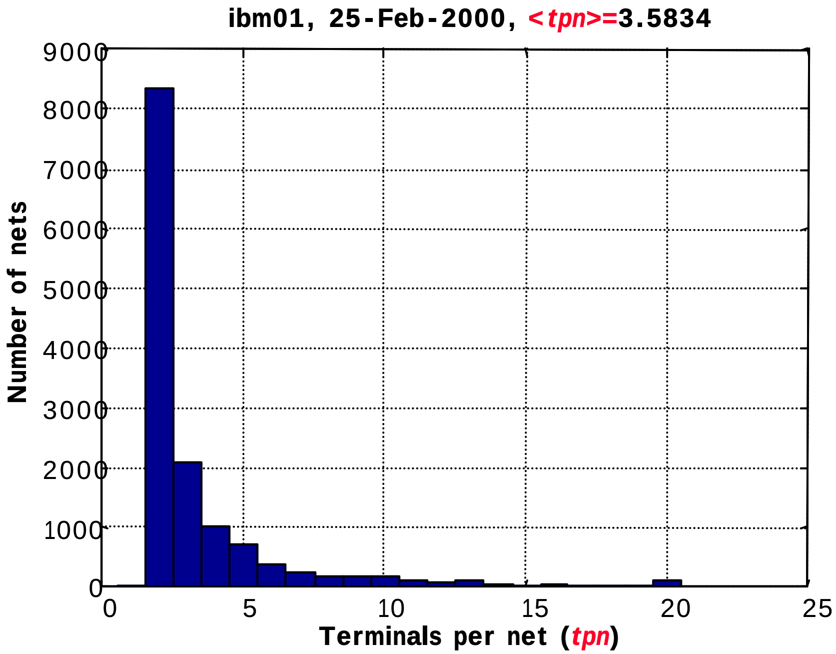
Net 2 has 3 terminals per net (*tpn*)

Cell 3 has 3 nets per cell (*npc*)

Netlists and signatures

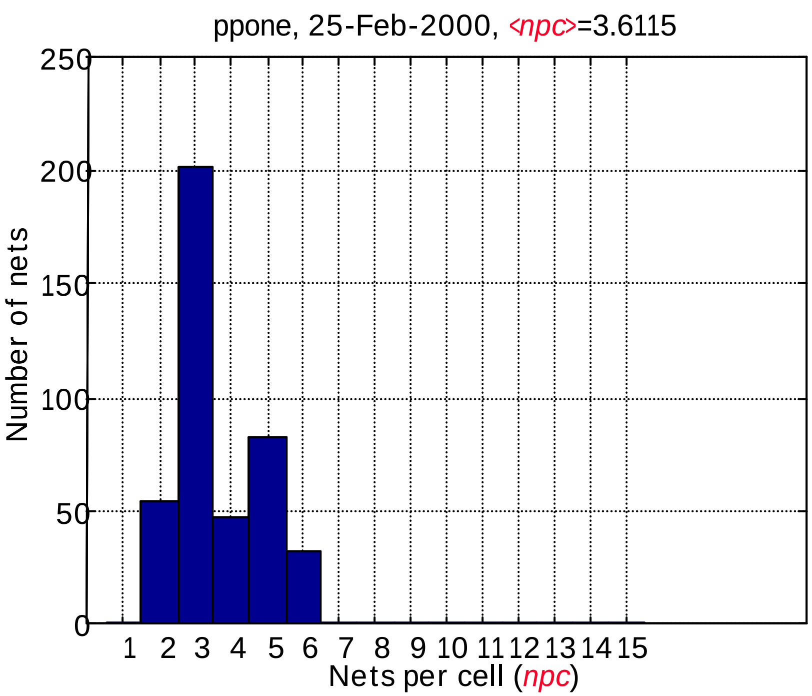
terminals per net (*tpn*)

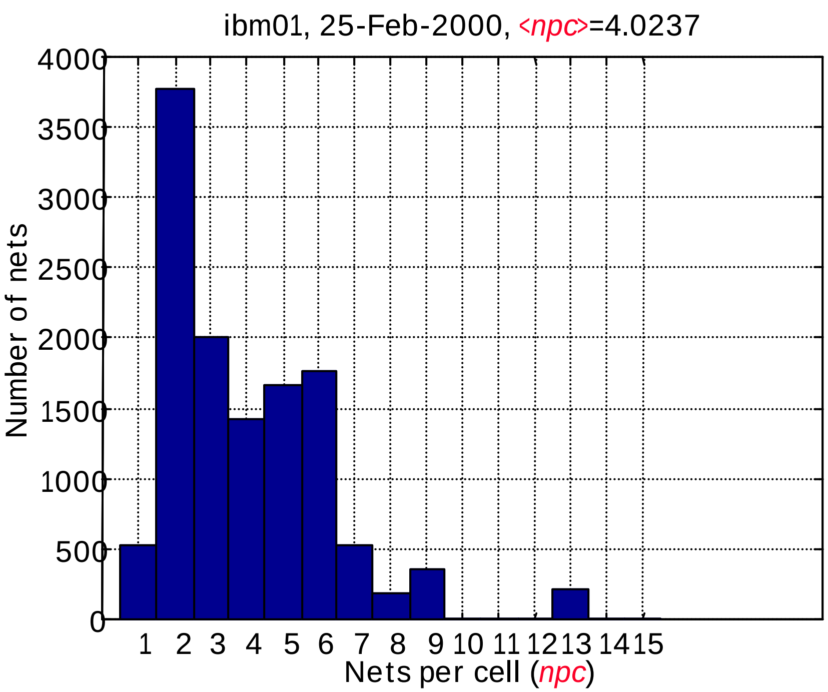




Netlists and signatures

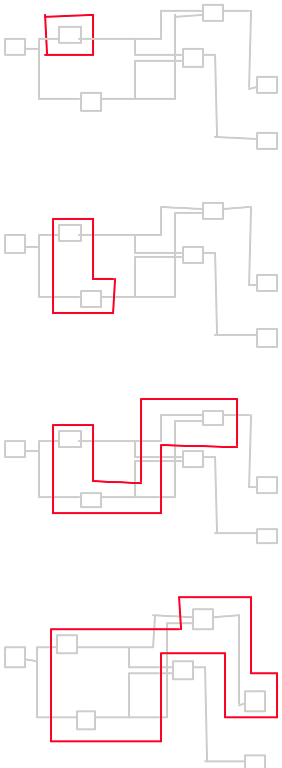
Nets per cell (*npc*)

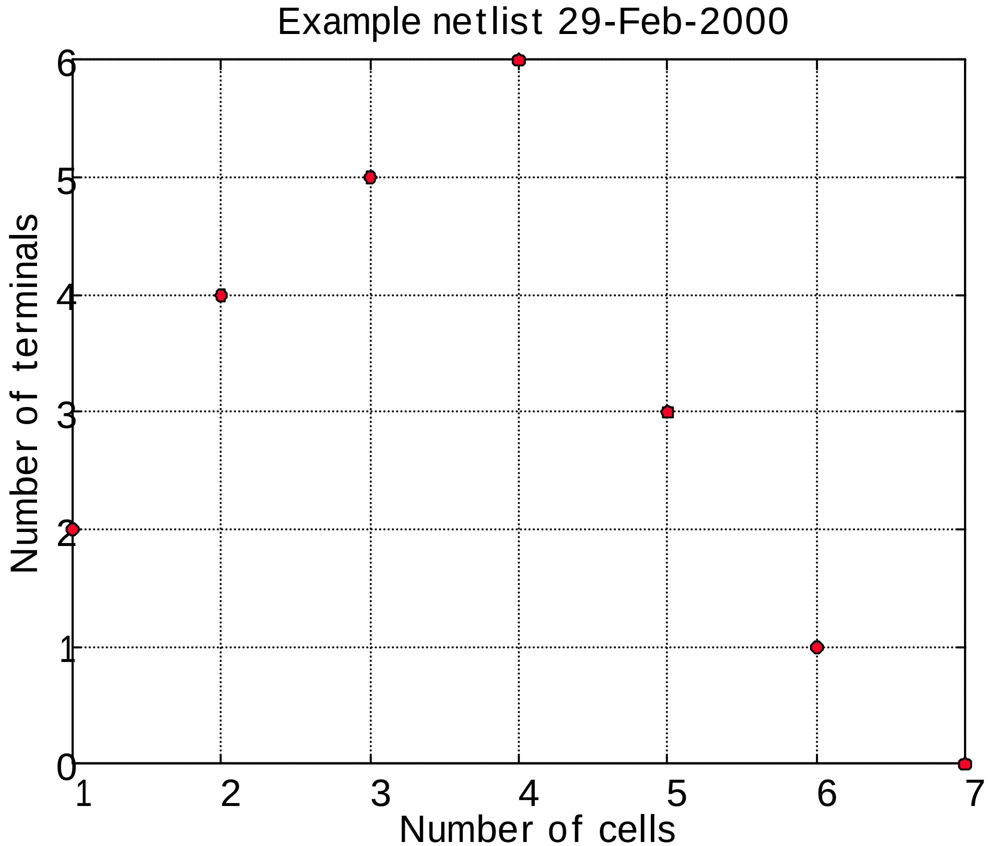




Netlists and signatures

Terminal counting





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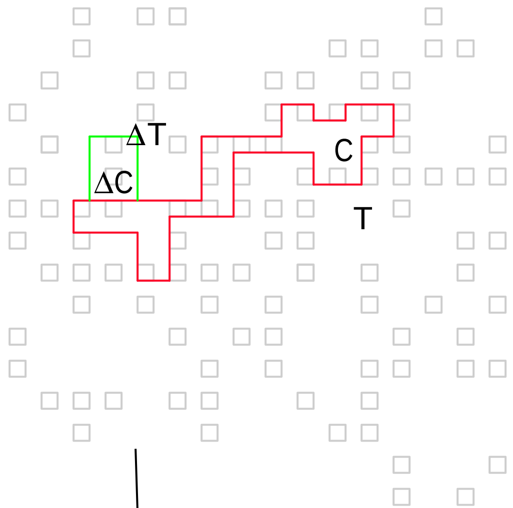
Netlists and signatures

Rent's rule



Statistically homogenous system

If an additional ΔC cells are added, what is the increase in terminals ΔT?
In the absence of any other information we might guess that

$$\Delta T = \left(\frac{T}{C}\right)\Delta C$$

But this is an overestimate since many of these ΔT terminals may already connect into larger red structure and so do not contribute to the total.

We introduce a factor r ($0 < r < 1$) which indicates how self connected the netlist is

$$\Delta T = r\left(\frac{T}{C}\right)\Delta C$$

Or, if ΔC, ΔT are small compared with C and T

$$\frac{dT}{T} \approx r\left(\frac{dC}{C}\right)$$

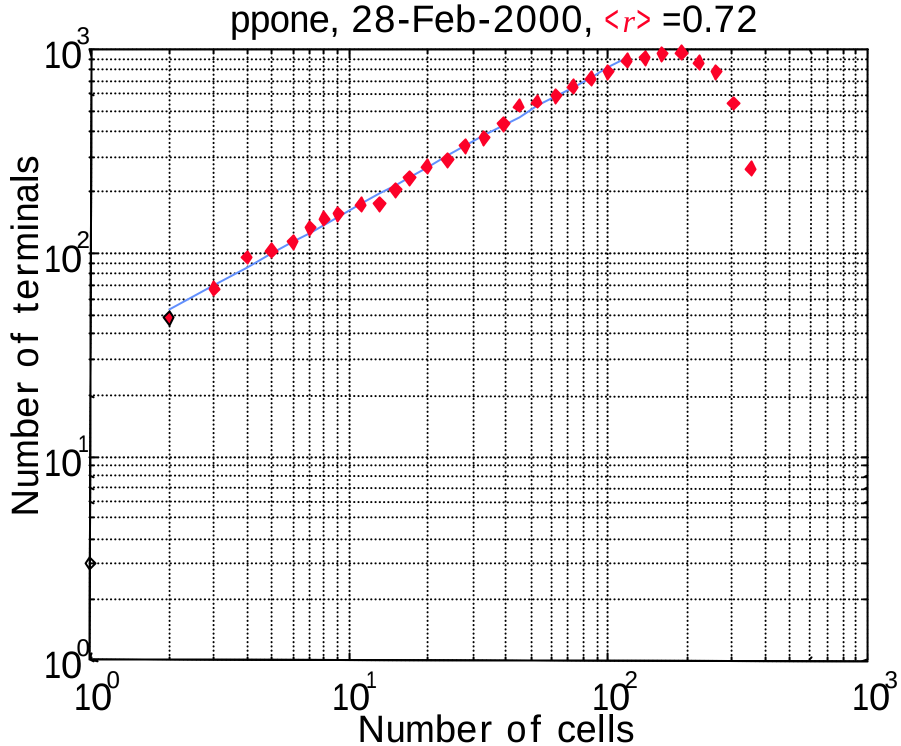
Which may be solved to yield

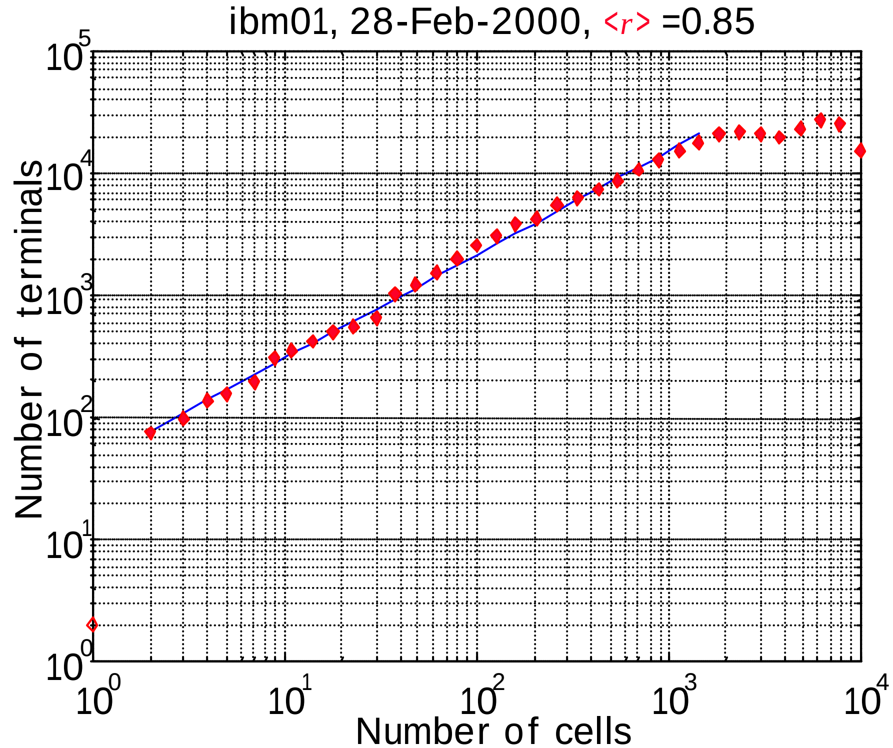
$$T = \langle npc \rangle\, C^{r}$$

Where $\langle npc \rangle$ is the average number of nets per cell, and is generated as a constant of integration

Netlists and signatures

Rent exponents





Netlists and signatures

Synthetic netlists

RMC (Random Mapped Circuit
Darnauer and Dai
- Top-down recursive partitioning
- Allocation based on Rent's rule

CIRC and GEN
Hutton, Rose, Grossman, and Corneil
- CIRC is an parameter profiler used as input for GEN
- Sequential circuits generated by gluing combinational circuits
- Not Rent-based

GNL (Generate NetList)
Stroobandt, Depreitere, and van Campenhout
- Bottom-up clustering approach
- allocation based on Rent's rule
- Sequential circuits possible

Signature invariant mutants
Brgles
- Generated my mutation of real circuits
- mutation maintains wiring signature invariance
- Rent's rule observed

PartGen
Pistorius, Legai, and Monoux
- Two-level hierarchical netlist generator
- first level selects from 4 standard circuits
- second level generates controller logic

Random transformations
Iwama, Hino, Kurokawa, and Sawada
- Starts with fixed input NAND gates
- Uses set of 12 transformations to generate any k-NAND functionally equivalent circuit

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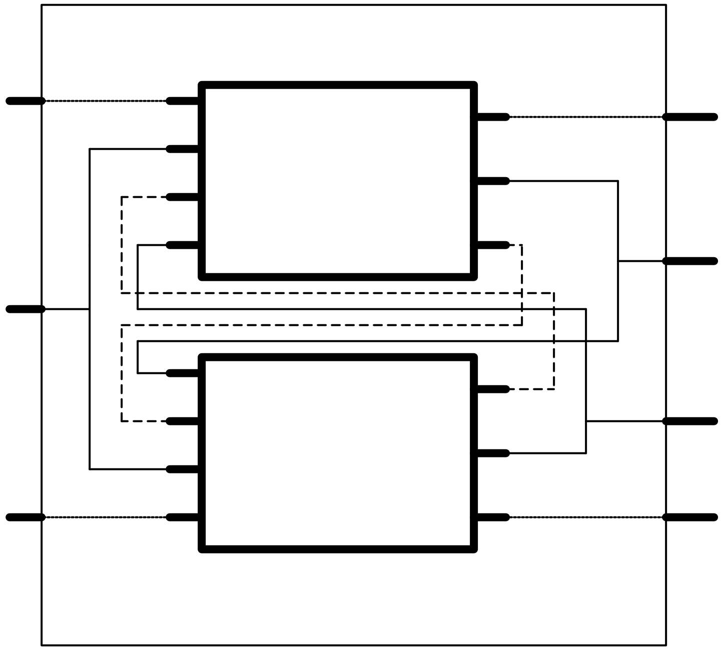
Netlists and signatures

Automatic netlist generation-GNL



- Number of logic blocks and number of inputs/outputs specified by user
- Logic blocks are paired and (pseudo)-random connections made between blocks as determined by Rent's rule.
- Constant ratio of internal to external connections at each level

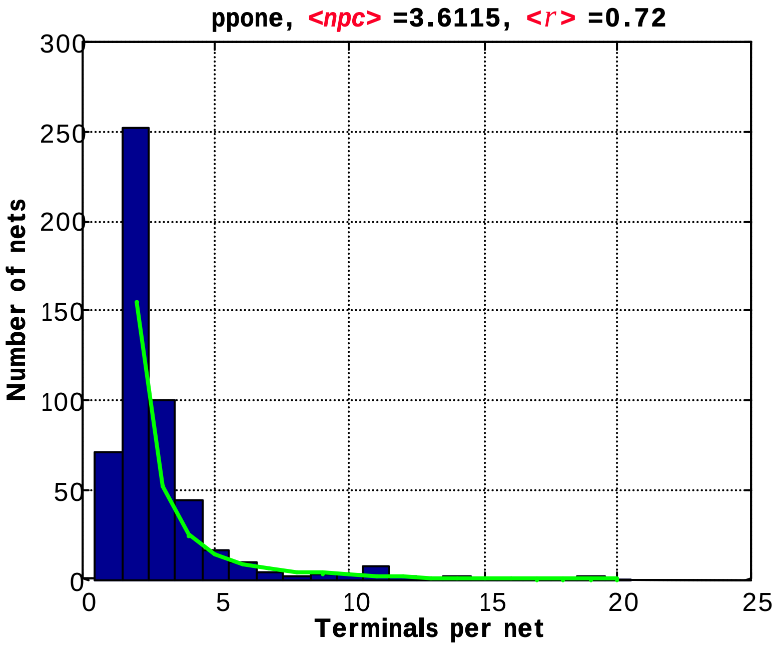
Generates a guaranteed Rent exponent and a realistic *tpn* distribution

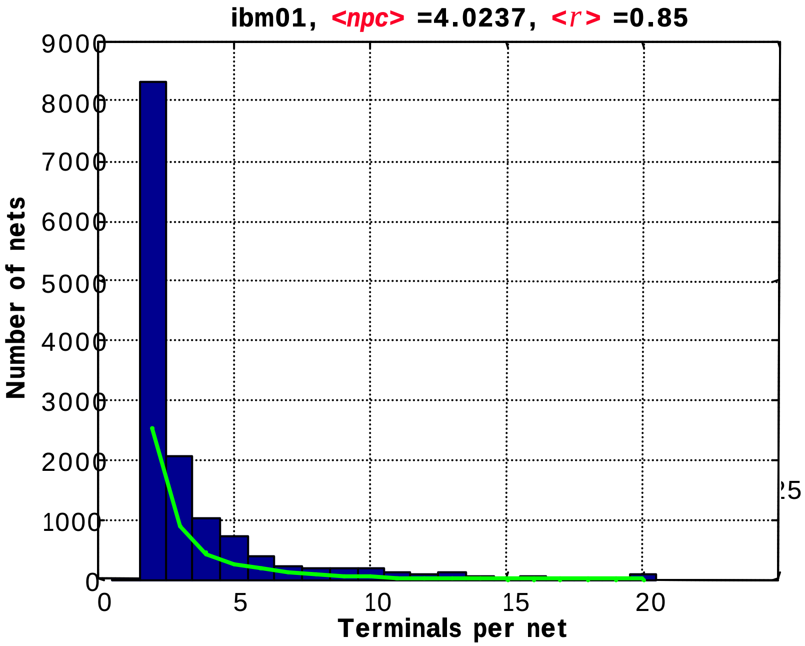
Netlists and signatures

parameter independence

Recent paper shows <*tpn*>, <*npc*> and <*r*> are not independent

$$tpn(m) = \langle npc \rangle C_{total} \left((m-1)^{\langle r \rangle - 1} - m^{\langle r \rangle - 1} \right) / m$$





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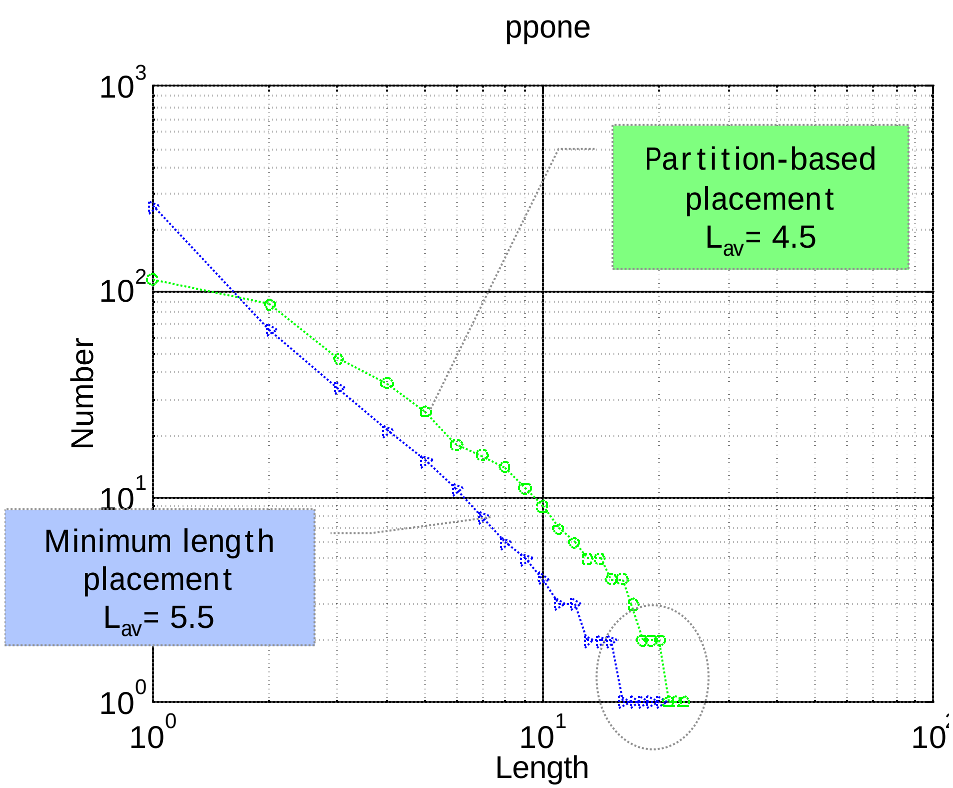
Netlists and signatures

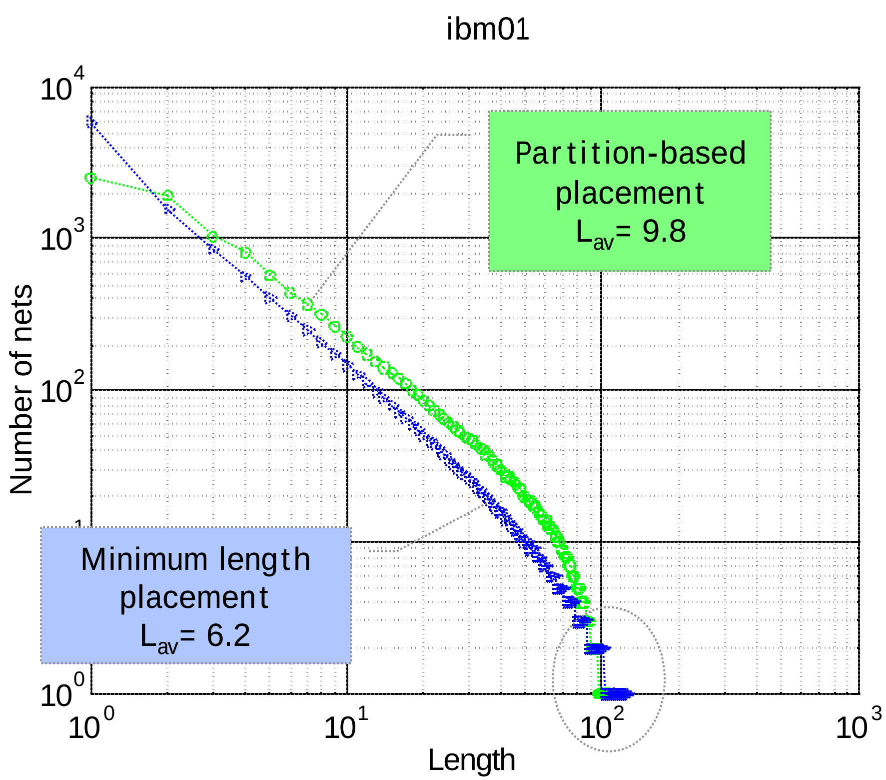
Summary

- <*tpn*> characterizes net fan-out
- <*npc*> characterizes cell fan-out
- <*r*> is the Rent exponent whose meaning is open for discussion.
- These parameters may not be independent
- What happens when we embed the netlist into a two-dimensional surface?

Partitioning and placement

Sample calculation





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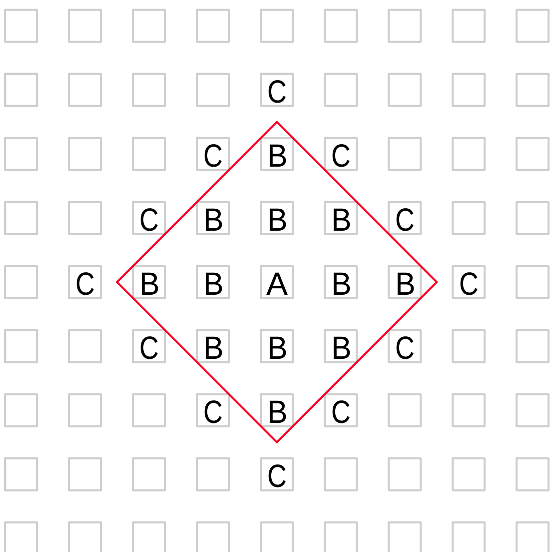
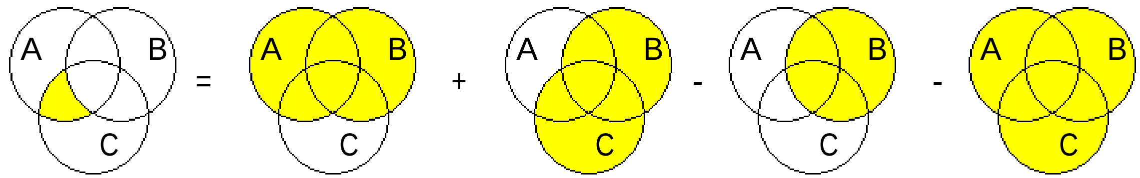
Partitioning and placement

Estimation of length distribution function *N(l)*





$$T_{A \circledR C} = T_{AB} + T_{BC} - T_{B} - T_{ABC}$$

Assumption: net cannot connect A,B, and C

$$T_{AB} = \langle npc \rangle (1 + C_B)^{\langle r \rangle} \qquad T_{BC} = \langle npc \rangle (C_B + C_C)^{\langle r \rangle}$$

$$T_{B} = \langle npc \rangle C_B^{\langle r \rangle} \qquad T_{ABC} = \langle npc \rangle (1 + C_B + C_C)^{\langle r \rangle}$$

Partitioning and placement

Conservation of terminals

$$T_{A\to C} = \langle npc \rangle \left[(1 + C_B)^{\langle r \rangle} + (C_B + C_C)^{\langle r \rangle} - C_B^{\langle r \rangle} - (1 + C_B + C_C)^{\langle r \rangle} \right]$$

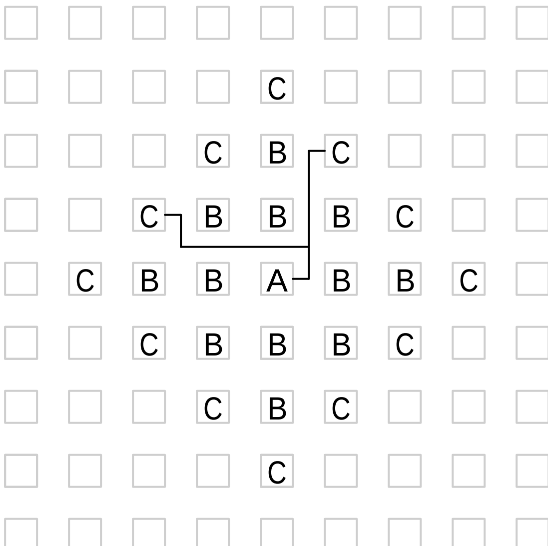
We now convert from the number of terminals to the number of nets using <*tpn*>

$$n_{A\to C} = T_{A\to C} / \langle tpn \rangle$$

$$n_{A\to C} = \frac{\langle npc \rangle}{\langle tpn \rangle} \left[(1 + C_B)^{\langle r \rangle} + (C_B + C_C)^{\langle r \rangle} - C_B^{\langle r \rangle} - (1 + C_B + C_C)^{\langle r \rangle} \right]$$

Assumptions: <u>all</u> nets have <*tpn*> terminals per net
<u>all</u> cells have <*npc*> nets per cell
<u>all</u> terminals in net lie in region A or C

Partitioning and placement
Embedding process (infinite 2D plane)



For cells placed in infinite 2D plane

$$C_C = 4l$$

$$C_B = \sum_{l=1}^{l-1} 4l = 2l(l-1)$$

$$n_{A\to C} = \frac{\langle npc\rangle}{\langle tpn\rangle}\left[\left(1+2l(l-1)\right)^{\langle p\rangle} + \left(2l(l-1)+4l\right)^{\langle p\rangle} - \left(2l(l-1)\right)^{\langle p\rangle} - \left(1+2l(l-1)+4l\right)^{\langle p\rangle}\right]$$

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Partitioning and placement
Reality check

(1) All cells have <*npc*> nets per cell
(2) All nets have <*tpn*> terminals per net
(3) Net cannot connect A,B, and C
(4) All terminals of net lie in region A or C

(2) is only consistent with (3) and (4) if <*tpn*> $= 2$, then $n_{A \circledR C} = n(l)$ and represents the number of 2-terminal nets of length l associated with a single cell

$$n(l) = \frac{\langle npc \rangle}{\langle tpn \rangle}\left[(1+2l(l-1))^{\langle r \rangle} + (2l(l-1)+4l)^{\langle r \rangle} - (2l(l-1))^{\langle r \rangle} - (1+2l(l-1)+4l)^{\langle r \rangle}\right]$$

For <*tpn*> > 2, $n(l)$ is internally inconsistent

Partitioning and placement

Probability function (infinite 2D plane)

We note

$$\sum_{l=1}^{\infty}\left(1+2l(l-1)\right)^{\langle r\rangle}+\left(2l(l-1)+4l\right)^{\langle r\rangle}-\left(2l(l-1)\right)^{\langle r\rangle}-\left(1+2l(l-1)+4l\right)^{\langle r\rangle}=1$$

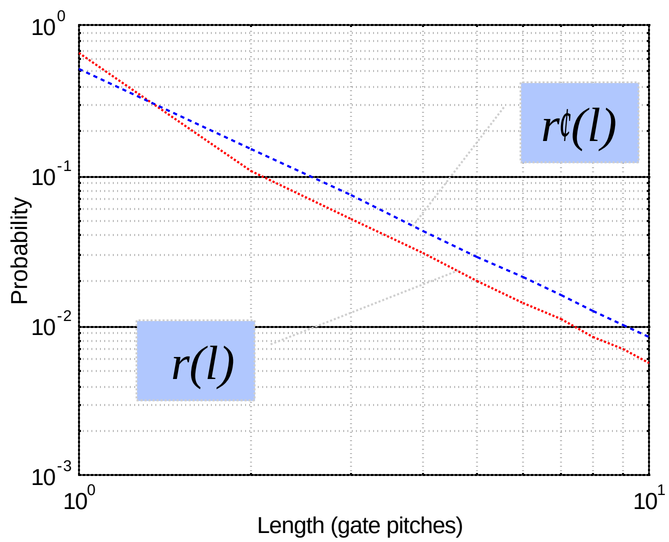
And so we can write

$$n(l)=\frac{\langle npc\rangle}{\langle tpn\rangle}r(l)$$

Where $r(l)$ is the probability that a cell has a 2-terminal net of length l.

Partitioning and placement

Approximate form for *r(l)* (infinite 2D plane)



By expanding individual terms in *r(l)* as binomial series we observe the underlying form

$$r'(l) \approx K\, l^{-(3-2\langle r \rangle)}$$

Where K is determined by the requirement that

$$1 = \sum_{l=1}^{\infty} K\, l^{-(3-2\langle r \rangle)}$$

And so we may write

Riemann zeta function

$$K = \frac{1}{x\left(3-2\langle r \rangle\right)}$$

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Partitioning and placement

Site densities and occupancies (infinite 2D plane)

In this context $r(l)$ is interpreted as the probability that a cell has a net of length l. We factor it into two parts

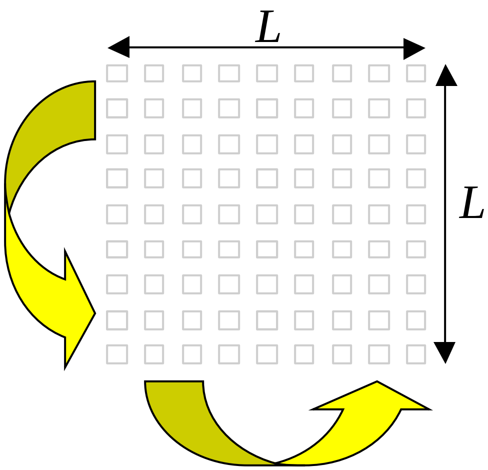
$$r(l) = q(l)\,4l$$

where $4l$ is the number of available wire sites per cell of length l and $q(l)$ is the expectation number of nets occupying that site. Since $q(l)$ can never be greater than 1, it may also be interpreted as an occupation probability

$$q(l) = \frac{1}{4l}\left[\left(1+2l(l-1)\right)^{\langle r\rangle} + \left(2l(l-1)+4l\right)^{\langle r\rangle} - \left(2l(l-1)\right)^{\langle r\rangle} - \left(1+2l(l-1)+4l\right)^{\langle r\rangle}\right]$$

Partitioning and placement

Planar model A



$$D_a(l) = 2lL^2$$

Finite system, $C_{tot}=L^2$, no edges, approximate form for $q'(l)$

$$N(l) = N_{tot}\, r'(l)$$

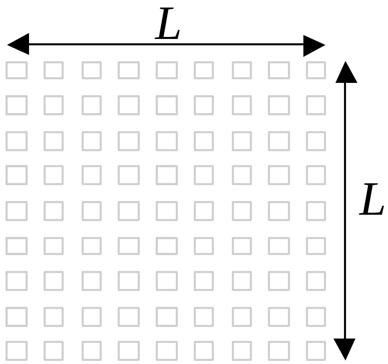
$$N_{tot} = \frac{\langle npc \rangle}{\langle tpn \rangle}\left(C_{tot} - C_{tot}^{\langle r \rangle}\right)$$

Assume $q'(l)$ retains functional form from infinite plane but now use site density function for finite cyclic system and appropriate normalization

$$r'(l) = K\, D_a(l)\, q'(l) \qquad 1 = \sum_{l=1}^{2L} r'(l)$$

Partitioning and placement

Planar model B



Finite system, $C_{tot}=L^2$, includes edge effects, use $q(l)$

$$N(l) = N_{tot}\, r(l)$$

$$N_{tot} = \frac{\langle npc \rangle}{\langle tpn \rangle}\left(C_{tot} - C_{tot}^{\langle r \rangle}\right)$$

$$D_b(l) = \begin{cases} l\left(l^2 - 1 + 6L(L-l)\right)/3 & \text{for } 1 \le l \le L \\ (2L-l+1)(2L-l)(2L-l-1)/3 & \text{for } L \le l \le 2L \\ 0 & \text{else} \end{cases}$$

Assume $q(l)$ retains functional form from infinite plane but now use site density function $D_b(l)$ and appropriate normalization

$$r(l) = K\, D_a(l)\, q(l) \qquad 1 = \sum_{l=1}^{2L} r(l)$$

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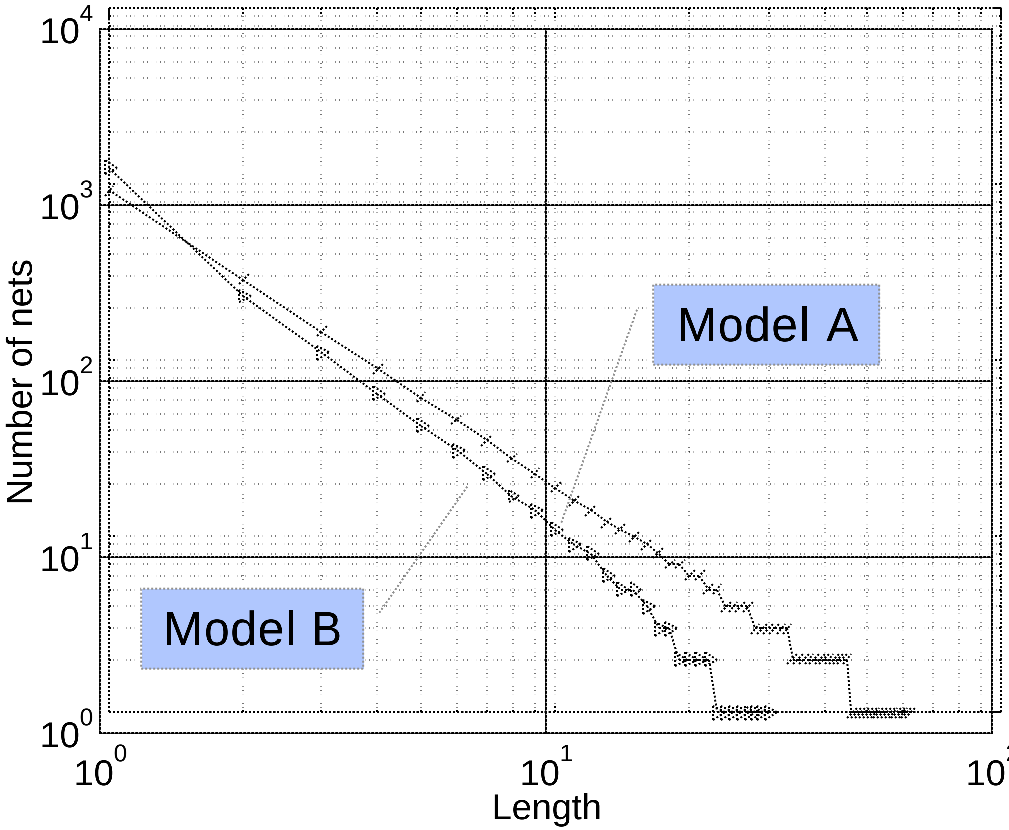
Partitioning and placement

Planar model comparison

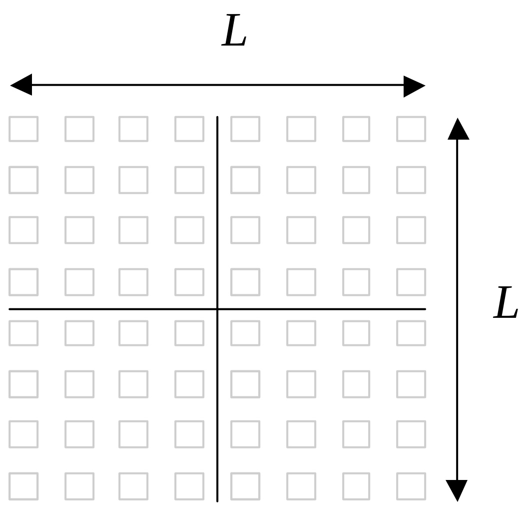
C_{tot} = *1024*
<tpn> = 2
<*npc*> = *4*
<*r*> = *0.66*

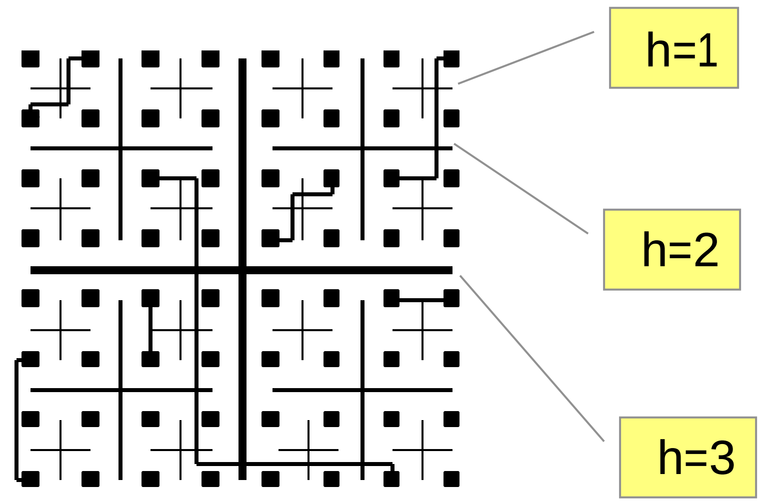
Model A: L_{av} = *4.53*
Model B: L_{av} = *2.27*



Partitioning and placement

Hierarchical model C



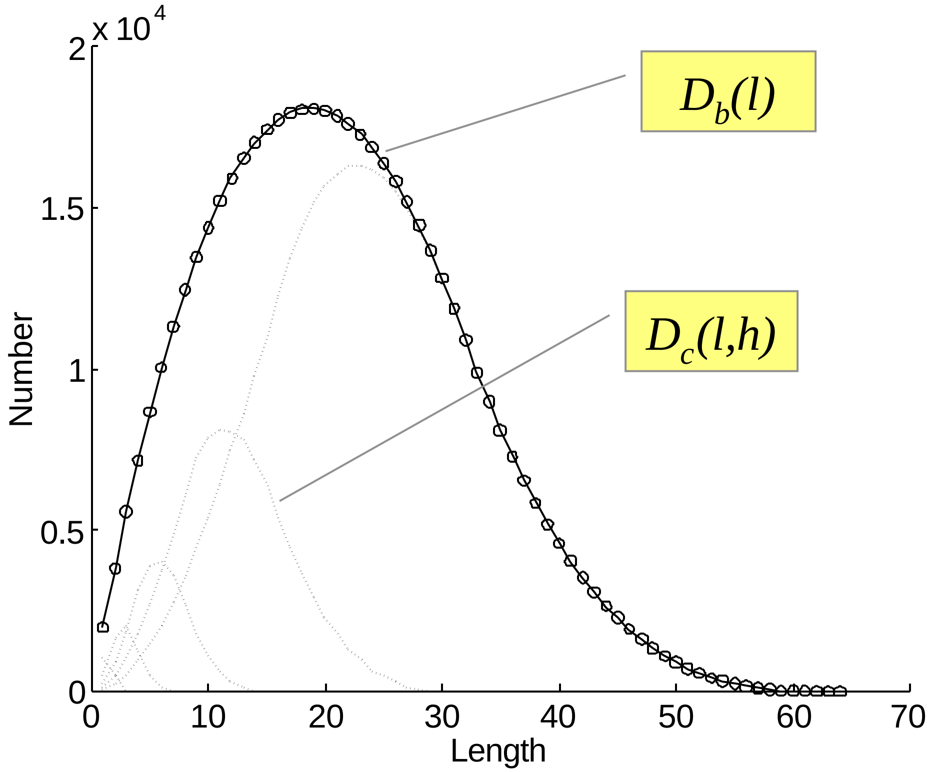


$$D_c(l) = \begin{cases} l(l(2L-l)+1) & \text{for } 1 \le l \le L/2 \\ \left(4L(1-L^2)-5l(1-l^2)+18Ll(L-l)\right)/3 & \text{for } L/2 \le l \le L \\ (2L-l+1)(2L-l)(2L-l-1)/3 & \text{for } L \le l \le 2L \\ 0 & \text{else} \end{cases}$$

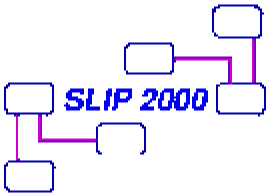
At level h there are $4^{(H-h)}$ equivalent partitions of side $L_h = 2^h$

Partitioning and placement

Relationship between Model B and C



$$D_b(l) = \sum_{h=1}^{H} 4^{H-h} D_c(l,h)$$



Partitioning and placement
Intra-layer model C

As before, within each level

$$N(l,h) = N_{h_{tot}} r(l,h)$$

where

$$r(l,h) = K\, D_c(l,h)\, q(l) \qquad 1 = \sum_{l=1}^{2L_h} r(l,h)$$

Net distribution for system is given by sum over hierarchies

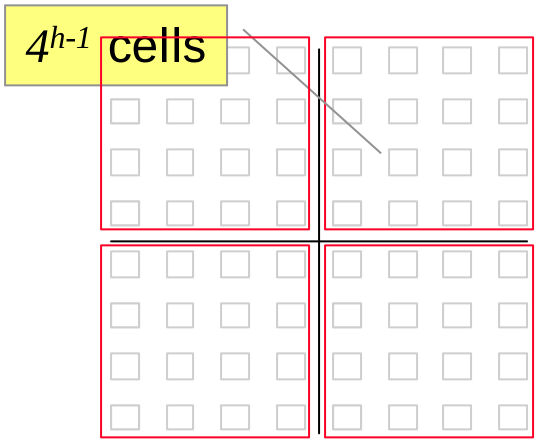
$$N(l) = \sum_{h=1}^{H} N(l,h)$$

Only remaining problem is to estimate number of nets in each level, N_{htot}

Partitioning and placement

Inter-layer model C

Number of nets in each layer may be determined by another application of Rent's rule. Consider single partition at level h



One group of 4^h cells generate $\frac{\langle npc \rangle}{\langle tpn \rangle} 4^{h\langle r \rangle}$ nets

Four groups of 4^{h-1} cells generate $4\frac{\langle npc \rangle}{\langle tpn \rangle} 4^{(h-1)\langle r \rangle}$ nets

Total number of nets in level h partition is

$$4\frac{\langle npc \rangle}{\langle tpn \rangle} 4^{(h-1)\langle r \rangle} - \frac{\langle npc \rangle}{\langle tpn \rangle} 4^{h\langle r \rangle}$$

Since there are 4^{H-h} equivalent partitions

$$N_{h_{tot}} = 4^{H-h}\left(4\frac{\langle npc \rangle}{\langle tpn \rangle} 4^{(h-1)\langle r \rangle} - \frac{\langle npc \rangle}{\langle tpn \rangle} 4^{h\langle r \rangle}\right)$$

Partitioning and placement
Hierarchical model D

Essentially same as Model C but with no intra-layer optimization. Then site occupancy probability is independent of length and equal to a constant, set $q(l)=K$, which is determined by normalization.

As before, within each level

$$N(l,h) = N_{h_{tot}} r(l,h)$$

where

$$r(l,h) = K\, D_c(l,h) \qquad\qquad 1 = \sum_{l=1}^{2L_h} r(l,h)$$

Net distribution for system is given by sum over hierarchies

$$N(l) = \sum_{h=1}^{H} N(l,h)$$

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Partitioning and placement

Model D average wire length

Simpler mathematical form for Model D enables rare analytical expressions

$$l_{h_{av}} = \sum_{l=1}^{2L_h} l\, r(l,h) = \frac{7}{9} 2^{h} - \frac{4}{9} 2^{-h}$$

$$l_{av} = \frac{1}{N_{tot}} \sum_{h=1}^{H} l_{av}\, N_{h_{tot}}$$

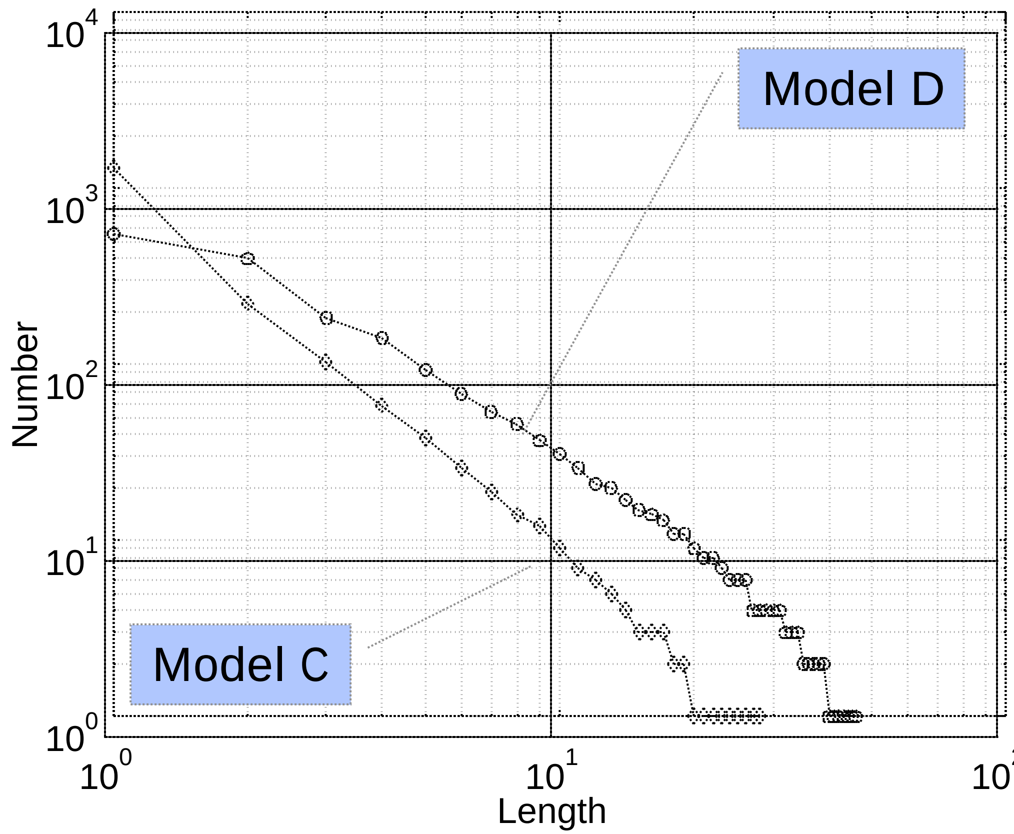
$$= \frac{2}{9} \frac{\left(1 - 4^{\langle r \rangle - 1}\right)}{\left(1 - C_{tot}^{\langle r \rangle - 1}\right)} \left\{ \frac{7\left(C_{tot}^{\langle r \rangle - 0.5} - 1\right)}{4^{\langle r \rangle - 0.5} - 1} - \frac{C_{tot}^{\langle r \rangle - 1.5} - 1}{4^{\langle r \rangle - 1.5} - 1} \right\}$$

Partitioning and placement

Hierarchical model comparison

C_{tot} = *1024*
<tpn> = 2
<npc> = *4*
<r> = 0.66

Model C: L_{av} = *2.05*
Model D: L_{av} = *5.14*



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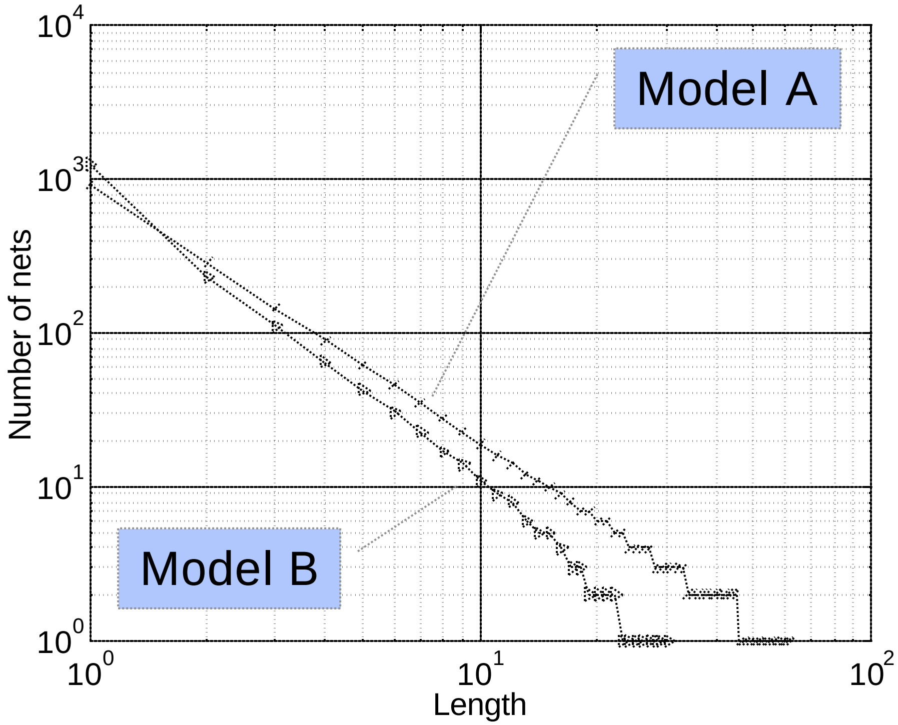
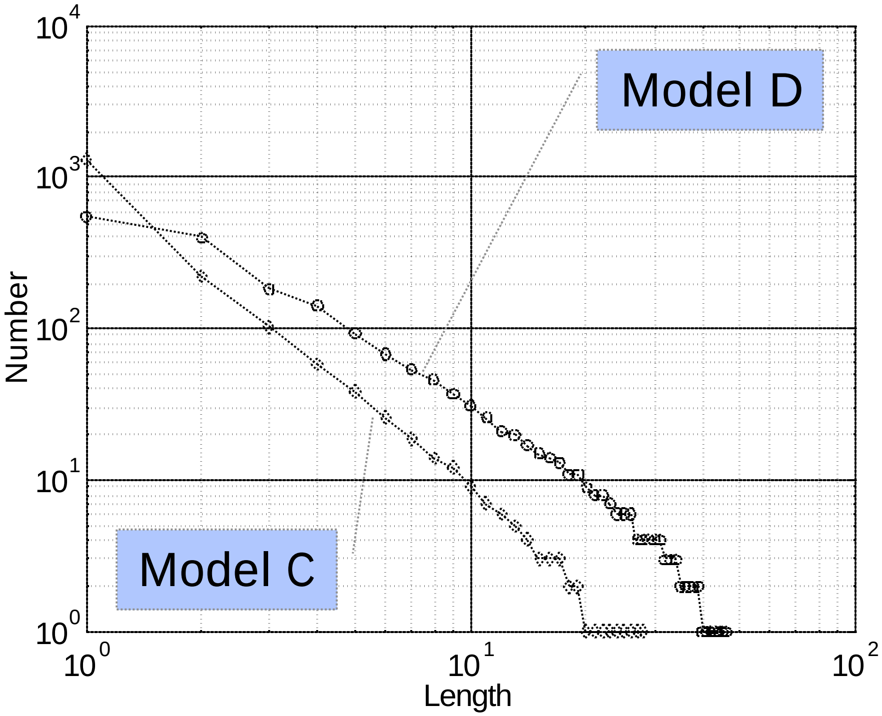
Partitioning and placement

Planar and hierarchical model comparison





Models B (planar) and C (hierarchical) are sometimes equivalent

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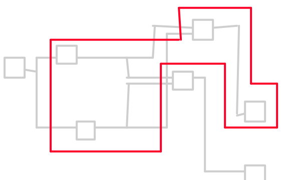
Rent exponents

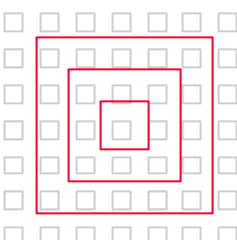
Topology versus Geometry

The first use of the Rent exponent was to estimate the distribution of m-pin nets



$$tpn(m) = \langle npc \rangle C_{total} \left((m-1)^{p_T - 1} - m^{p_T - 1} \right) / m$$

Topological Rent exponent, now written as p_T



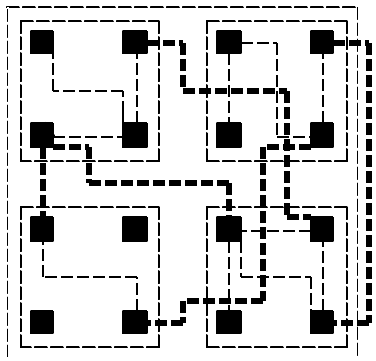
$$q(l) = \frac{1}{4l} \left[(1 + 2l(l-1))^{p_G} + (2l(l-1) + 4l)^{p_G} - (2l(l-1))^{p_G} - (1 + 2l(l-1) + 4l)^{p_G} \right]$$

Topological Rent exponent inappropriate. Define geometrical Rent exponent p_G. Measure of placement optimization, not an intrinsic netlist property

But how do we estimate the geometrical Rent exponent?

Rent exponents
Wiring cell analysis

Let us consider a simple two-level circuit, optimized for placement



With reference to N$_{htot}$ from inter-layer model C we note that

$$x = \frac{N_{(h+1)_{tot}}}{N_{h_{tot}}} = 4^{-(1-p_G)} \qquad \text{or} \qquad p_G = 1 + \log_4 x$$

also

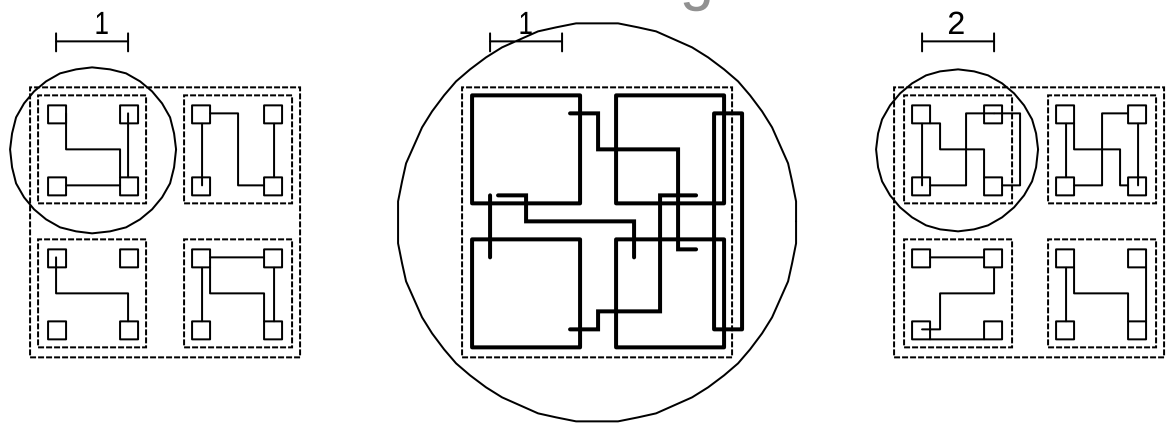
$$\langle npc \rangle_{1+2} = \langle tpn \rangle \frac{N_{1_{tot}} + N_{2_{tot}}}{16} = \frac{\langle tpn \rangle}{4^2} \sum_{h=1}^{2} N_{h_{tot}}$$

For the above example $N_{1tot}=11$, $N_{2tot}=5$ and <tpn> =2.0. Therefore

$$p_G = 0.431 \quad \langle npc \rangle_{1+2} = 2.0$$

Rent exponents

Dilation of wiring cell



Two level system

$$\langle npc\rangle_{1+2} = \frac{\langle tpn\rangle}{4^2}\sum_{h=1}^{2} N_{h_{tot}}$$

$$p_G = 1 + \log_4 x$$

H level system

$$\langle npc\rangle = \langle npc\rangle_{1+\cdots H} = \frac{\langle tpn\rangle}{4^H}\left[\sum_{h=1}^{2} N_{h_{tot}} + \sum_{h=3}^{H} N_{h_{tot}}\right]$$

$$p_G = 1 + \log_4 x$$

x is constant if p_G is constant

Rent exponents

Monte Carlo sampling

Therefore

$$N_{2_{tot}} = x\,N_{1_{tot}}\,,\; N_{3_{tot}} = x^2 N_{1_{tot}}\,,\; \text{etc.}$$

and so

$$\langle npc \rangle = \frac{\langle tpn \rangle N_{1_{tot}}}{4^H} \sum_{h=1}^{H} x^{h-1} = \langle npc \rangle_1 \frac{1-x^H}{1-x}$$

System defined if we know x and $<npc>_1$

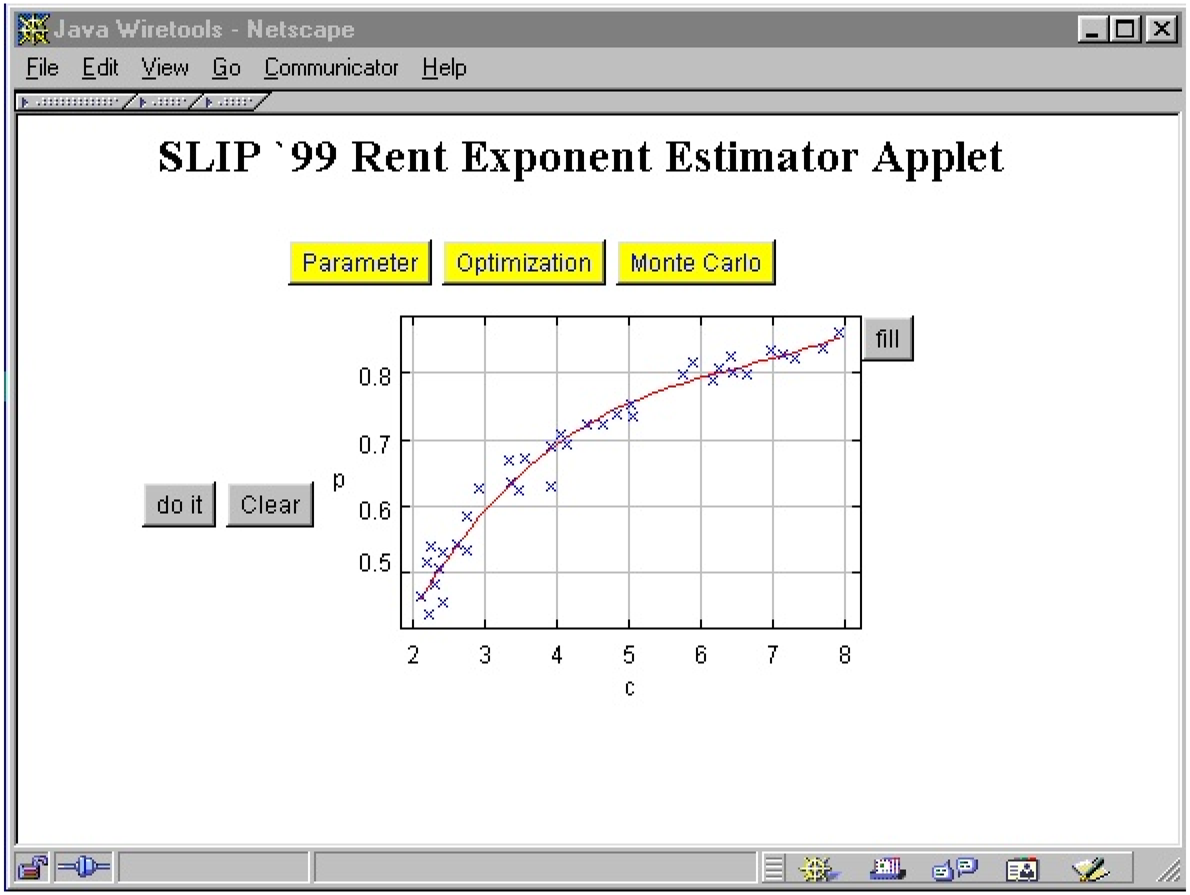
For the example wiring cell $x=0.455$ $<npc>_1=1.375$. For a circuit of size $C_{tot}=10^6$ ($H=9.966$)

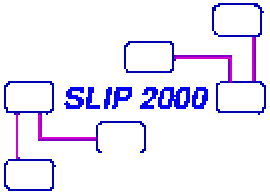
Known *a priori*

$p_G=0.431$ $<npc> =2.52$

Rent exponents

Sampling applet

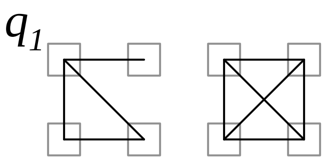




Rent exponents

Dilational filter

In calculating the Rent exponent we are only interested in details which are dilationally invariant.



Let probability that a single cell is connected to another cell at lowest level be q_1

Probability of there being a majority of nets within group of four cells is

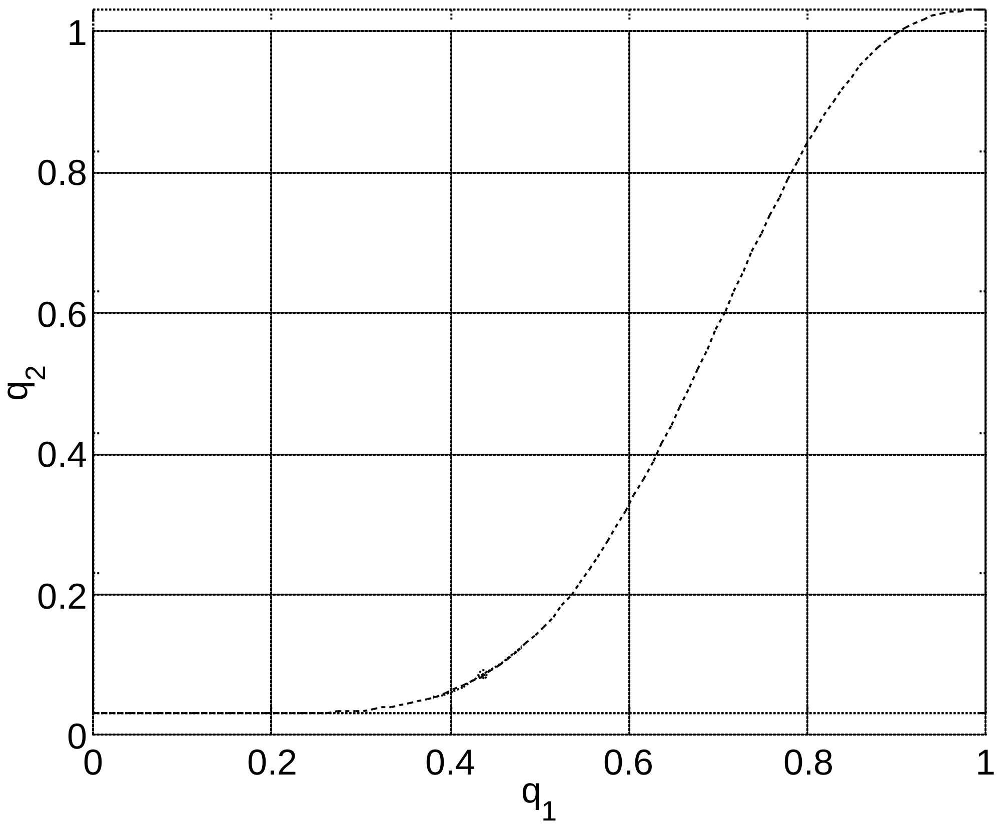
$$q_1^6 + 6q_1^5(1-q_1) + 15q_1^4(1-q_1)^2$$

Probability of connection between groups of four cells at level 2 is

$$q_2 = \Re[q_1] = \left[q_1^6 + 6q_1^5(1-q_1) + 15q_1^4(1-q_1)^2\right]^2$$

Rent exponents

Non-linear functionality



$$x = \frac{N_{2_{tot}}}{N_{1_{tot}}} = \frac{D_{2_{tot}}}{D_{1_{tot}}} \frac{q_2}{q_1}$$

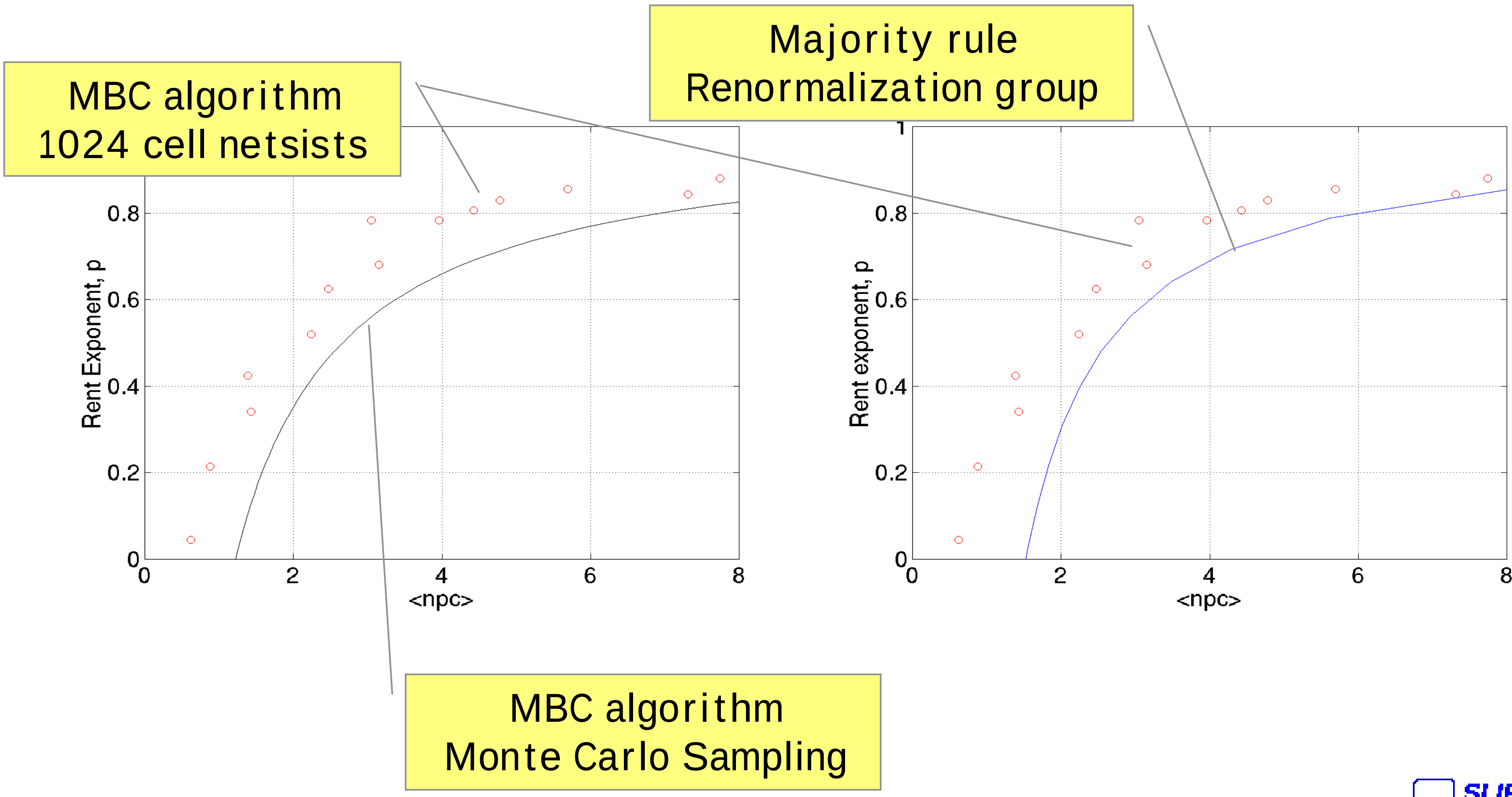
$$= 4 \frac{q_2}{q_1} = 4 \frac{\Re[q_1]}{q_1}$$

$$\langle npc \rangle_1 = \langle tpn \rangle \frac{6}{4} q_1$$

x and $<npc>$ expressed parametrically in terms of q_1

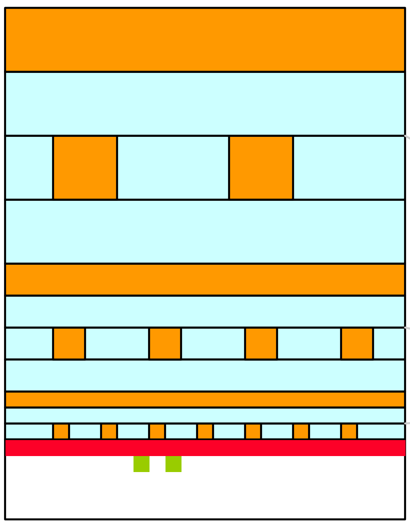
Rent exponents

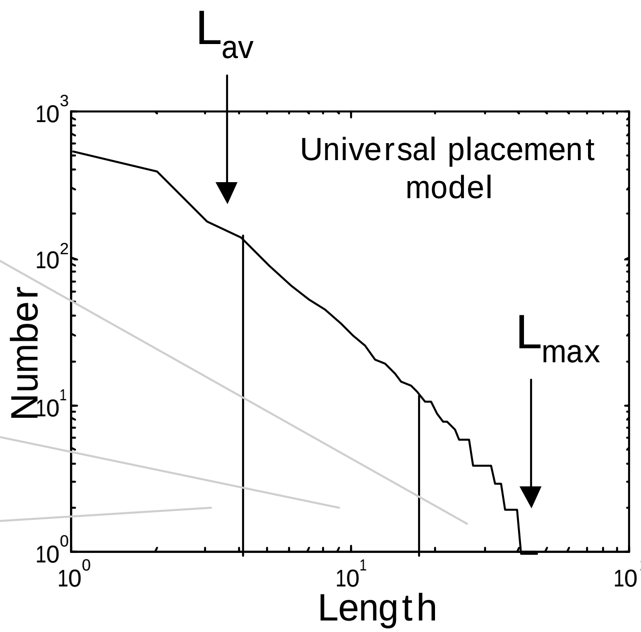
Theory versus experiment



What do you want to model today?

Cycle time





- Layer assignment
- Optimal repeater insertion
- Optimal power dissipation
- Effects of placement
- Effects of wiring signature

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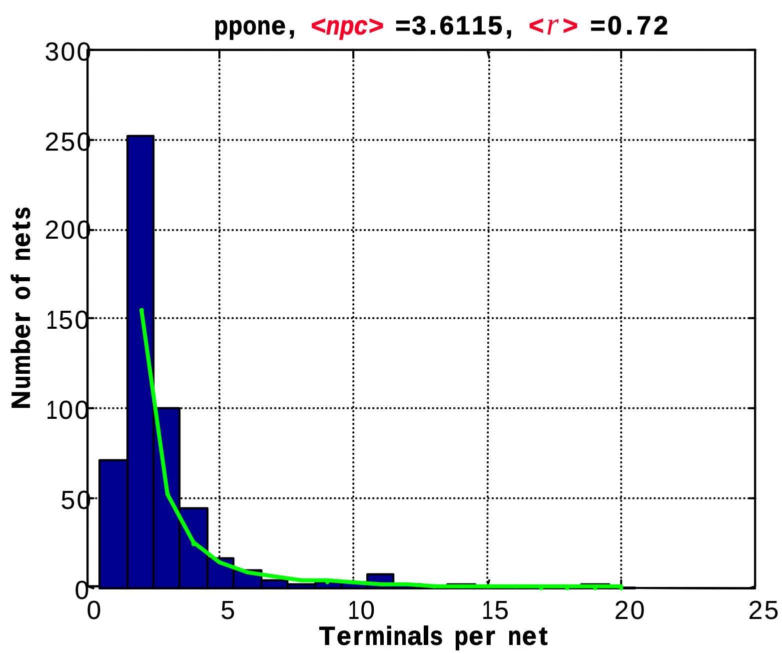
What do you want to model today?

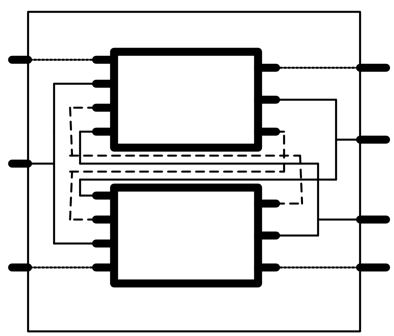
Wiring Signatures

$$tpn(m) = \langle npc \rangle C_{total} \left((m-1)^{\langle r \rangle - 1} - m^{\langle r \rangle - 1} \right) / m$$



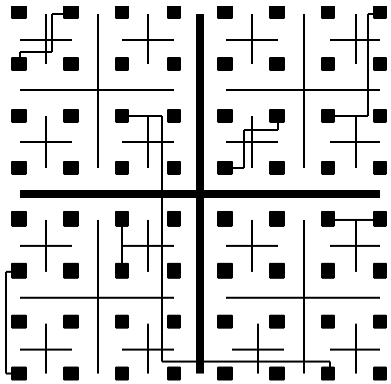
What are sufficient parameters to characterize netlists

$<tpn>$, $<npc>$, and p_T are not independent



What do you want to model today?

Universal placement model



$$p_G = p_G' \text{ Model B}$$

$$p_G \neq p_G' \text{ Model C}$$

$$p_G = 1 (q = K) \text{ Model D}$$

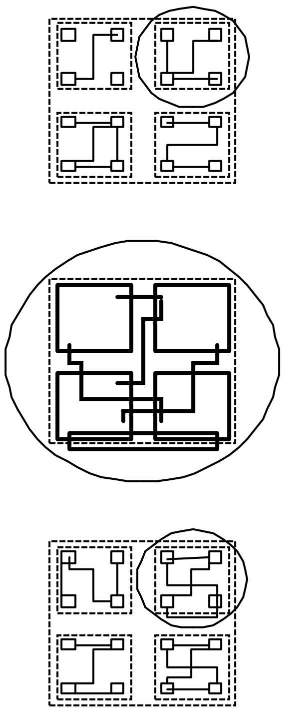
$$N(l) = K \sum_{h=1}^{H} N_{h_{tot}} D_c(l,h) q(l)$$

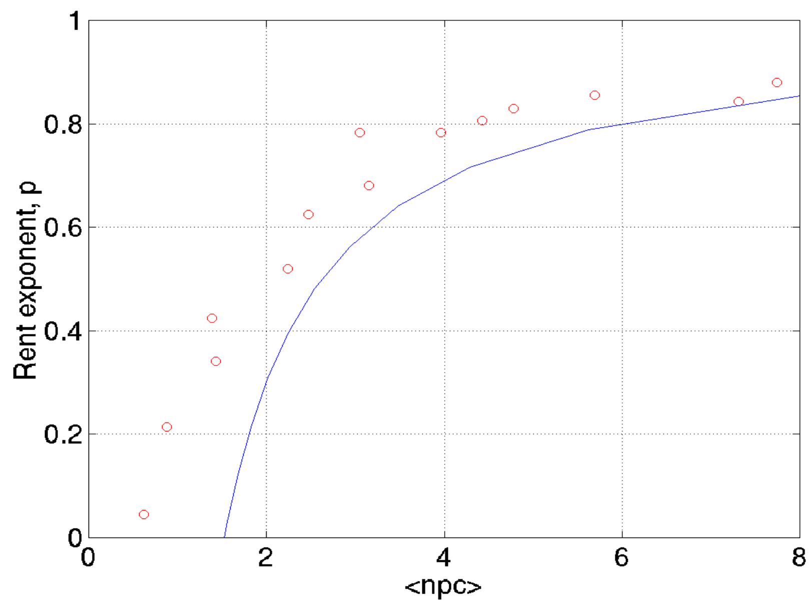
$$q(l) = \frac{1}{4l}\left[(1 + 2l(l-1))^{p_G} + (2l(l-1) + 4l)^{p_G} - (2l(l-1))^{p_G} - (1 + 2l(l-1) + 4l)^{p_G} \right]$$

$$N_{h_{tot}} = 4^{H-h}\left(4\frac{\langle npc \rangle}{\langle tpn \rangle} 4^{(h-1)p_G'} - \frac{\langle npc \rangle}{\langle tpn \rangle} 4^{hp_G'} \right)$$

What do you want to model today?

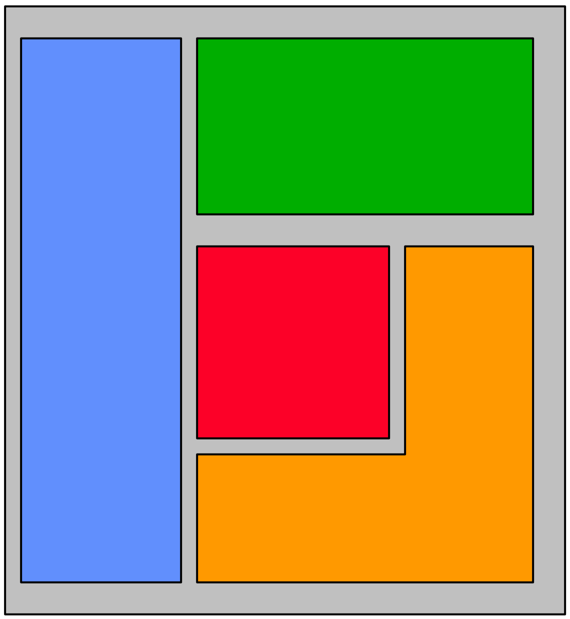
Rent exponents





What do you want to model today?

Heterogeneous systems



Object oriented approach to system-on-a-chip integration

Extremely difficult to predict interconnect resources required to implement global wiring between inhomogeneous system blocks

Global nets require different modeling techniques