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# Tutorial on Congestion Estimation

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# Outline

- Motivation
- WL prediction
  - Individual net
  - Total interconnect length
- Congestion prediction
- Concurrent wirelength and congestion prediction
- Density utilization alleviation
- Statistical timing analysis
- Integrated congestion and delay prediction
- Conclusion

# Motivation

- To achieve more efficient designs => combining front-end floorplanning and physical placement.
- The total wirelength affects three major parameters:
  - Chip size
  - Clock frequency
  - Power dissipation.
- Interconnect length is a measure of the quality of the placement
- accurate predictions of circuit properties are necessary to limit the vast search space

# Motivation

**New wirelength and congestion prediction methods are needed:**

- Increase in size and sophistication of circuits
- Decrease in physical feature size
- Presence of IP blocks
  - Soft IP
  - Hard IP
- Accurate prediction help improve
  - floorplanning
  - Placement
  - routing

# Integrated WL and Cong. Prediction

- Minimizing the total routed wirelength a fundamental goal
- In the presence of IP blocks, alleviating congestion after placement may result in an abrupt increase in wirelength
  - Congestion needs to be estimated early enough
- Internal routing demand is well-correlated with the length of interconnects
  - Congestion estimation needs the information of wirelength
- Detour around the congested area with no IP blocks.
  - Wirelength estimation may need to know the congestion map of the circuit
- Congestion and wirelength estimation are dependent

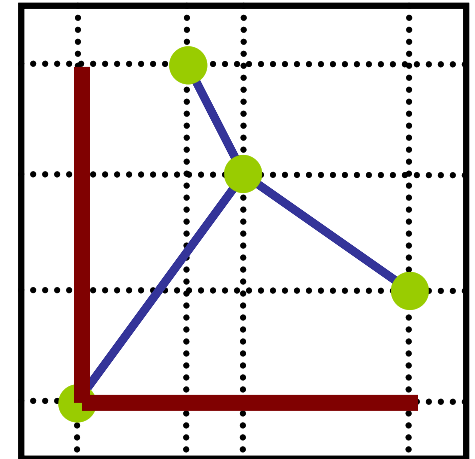
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# WL Estimation: Individual Net

## Half Perimeter Bounding Box

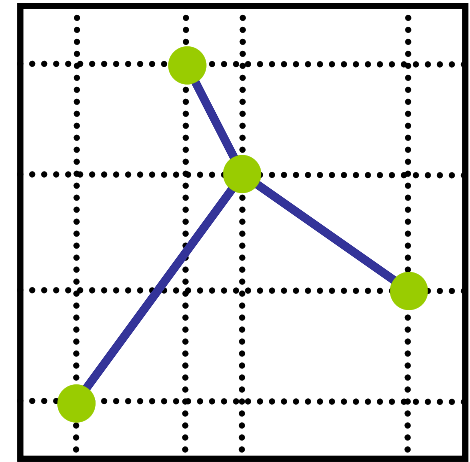
- Optimal solution for 2-pin and 3-pin nets
- Lower bound for nets with higher degree.
- However, it can significantly underestimate wirelength for higher-degree nets.
- Net weighting technique to scale up the HPWL estimation.



# WL Estimation: Individual Net

## Minimum Spanning Tree

- A spanning tree  $T = \{V_T, E_T\}$  of a graph is a sub-graph  $T$  of  $G$ , where  $T$  is a tree (no cycles) and such that  $V_T = V$  and  $E_T \subset E$ .
- MST = a minimum weight spanning tree over a weighted graph
- Can produce good wirelength estimation in reasonable amount of time.
- Best Time complexity =  $O(n \log(n))$
- Simple  $O(n^2)$  time implementation of Prim's algorithm is usually used

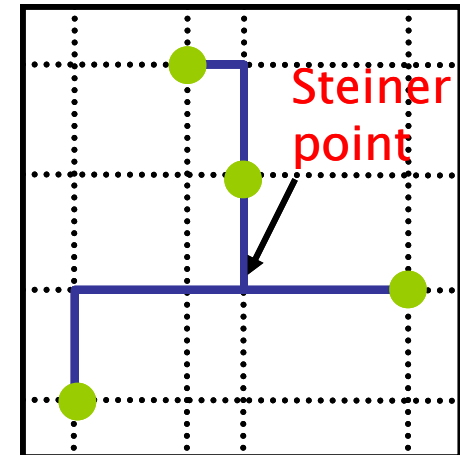




# WL Estimation: Individual Net

## Rectilinear Steiner Minimal Tree

- Given a set  $P$  of  $n$  points, find a set  $S$  of Steiner points such that  $MST(P \cup S)$  has the minimum cost.
- Optimal RSMT can always be constructed based on the Hanan grid.
  - the length of an edge in the Hanan grid is equal to distance
- RSMT is NP complete
- Good heuristic: Iterative 1-steiner RSMT

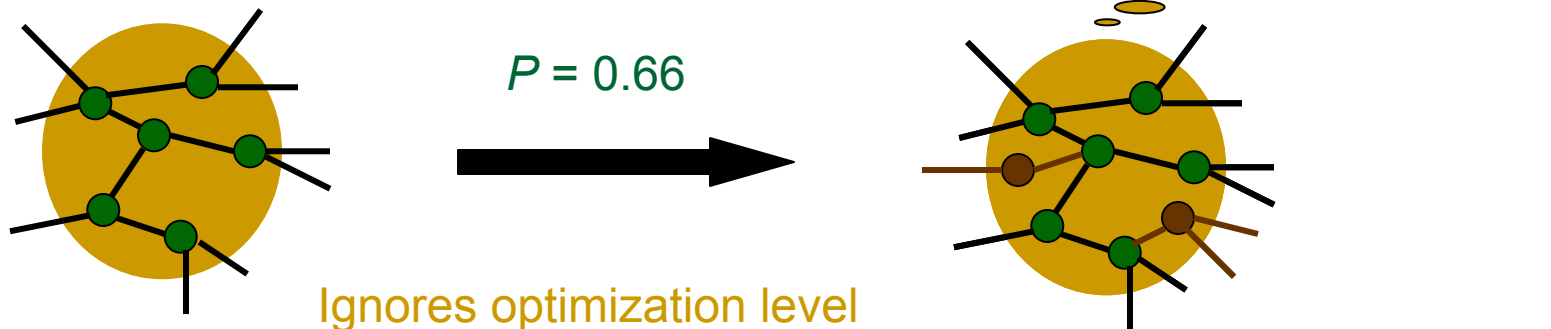


# Rent's Rule

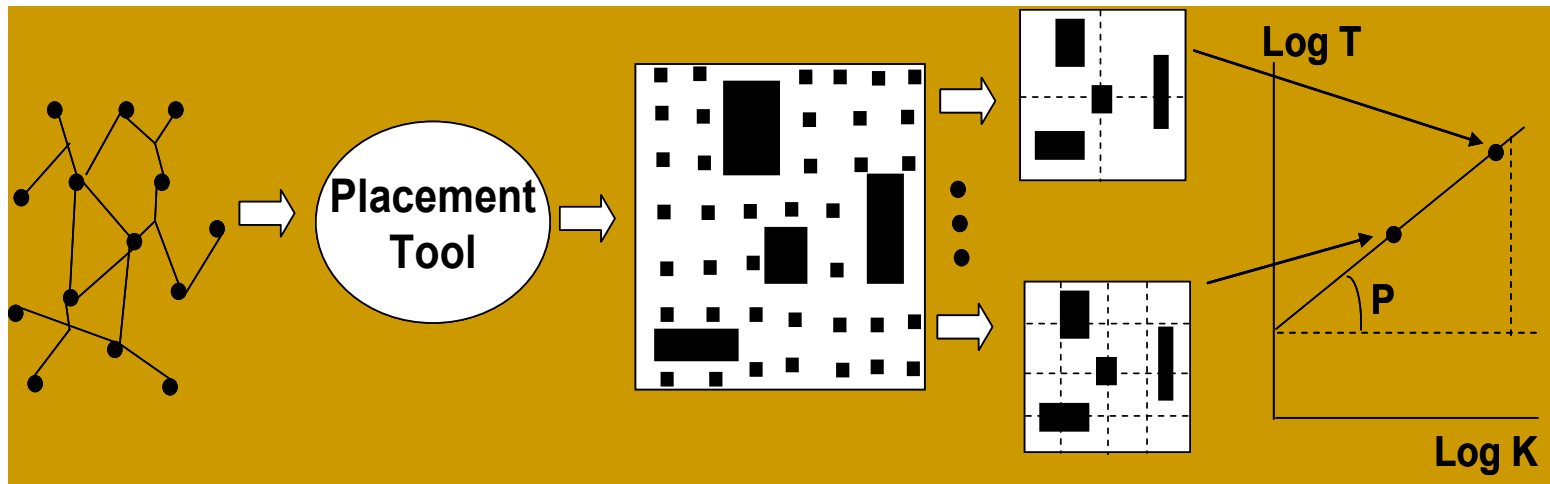
- Base for most of WL estimation methods
- Empirical power law,  $T=AC^P$
- $A$ = Rent coefficient
- $P$ = Rent exponent
  - Higher rent exponent  $\rightarrow$  Higher complexity
- Rent exponent range :  $0 \leq P \leq 1$ 
  - Regular arch. (RAM) :  $P = 0.5$
  - Complex arch. (VLSI circuits) :  $P = 0.75$
  - Random logic :  $P = 1$
  - Long chain of logic :  $P = 0$

# Rent's Rule

- $T$  : Terminals per partition
- $C$  : Cells per partition
  - $T/C$  Terminals per cell
    - $\Delta T = (T/C) \Delta C$
  - Small  $\Delta T, \Delta C$ 
    - $dT = (T/C) dC$
  - Results in  $T = AC$ 
    - $A = \text{Avg. no. of Terminals}$
- Placement optimization level
  - $T/C$  Terminals per cell
    - $\Delta T = P(T/C) \Delta C$
  - Small  $\Delta T, \Delta C$ 
    - $dT = P(T/C) dC$
  - Results in  $T = AC^P$



# Rent's Rule Extraction

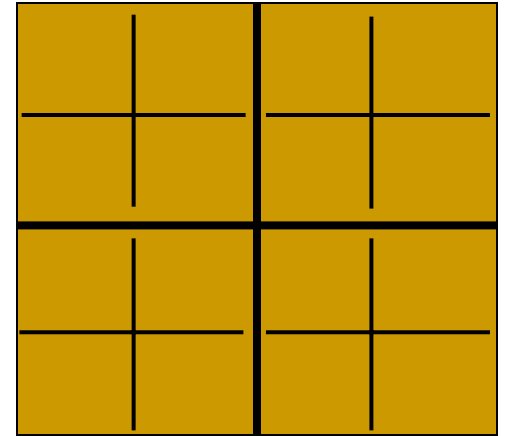


- Partitioning
- Coarse placement
- Log-Log plot of external terminals vs. avg. no. of cells
- Rent exponent: slope of the regression line

$$\log(T) = \log(A) + P \log(K)$$

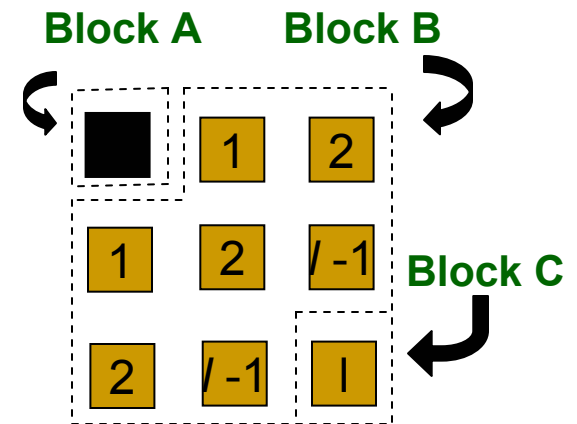
# Donath WL Estimation Method

- Hierarchical estimation
  - Partition into 4 sub-circuits.
  - At each level of hierarchy,
    - $n_h$ : Number of interconnections
    - $L_h$ : Average length of interconnections
  - Partitioning satisfies the Rent's rule.
- Drawback:
  - Ignores IP Blocks



# Davis WL Estimation Method

- Derivation of wirelength distribution
- Recursively applying Rent's rule
  - Stochastic WL distribution of a single gate
  - Remove it from the system
  - Repeat process for other gates
  - Superimpose WL distributions
- Drawbacks
  - Complex equations
  - Ignores IP Blocks



# Non-Uniform Probability Distribution

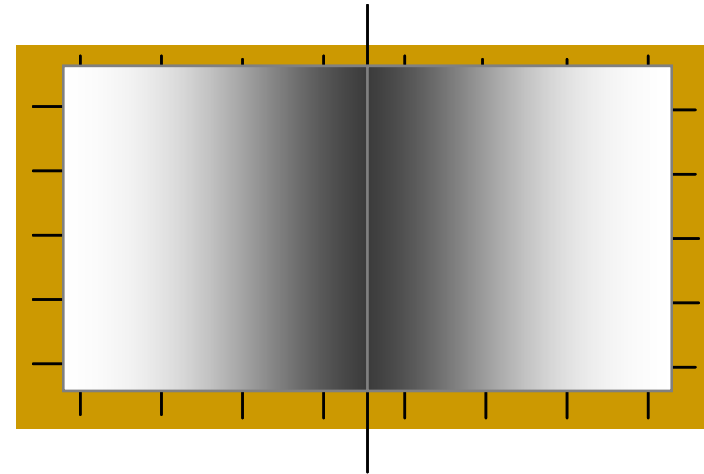
## Why Overestimate in Donath Method?

- Uniform probability distribution
- Optimal placement behavior
  - Keep connected cells closer to each other
  - More short wires than long wires
- Solution: Non-uniform probability distribution
  - Advantage: Accurate estimation
  - Disadvantage: Complex equations

# Non-Uniform Probability Distribution

- Donath hierarchical scheme
  - Treat each level of hierarchy independently
  - Uniform probability distribution for terminals
- Optimal placement behavior
  - More terminals at the border
- Define:
  - $q(l)$ : Occupying probability

$$q'(l) \approx l^{-(4-2P)}$$





# Non-Uniform Probability Distribution

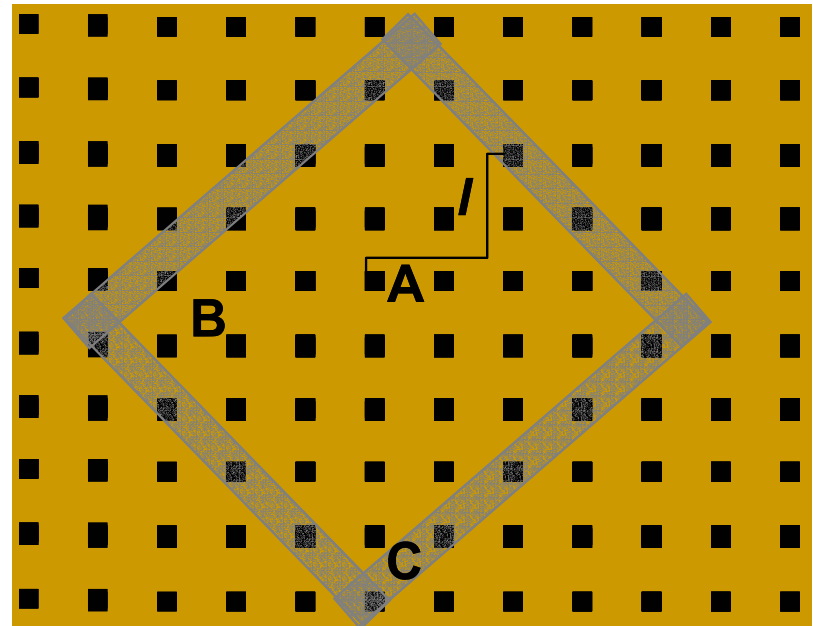
- Conservation of terminals
  - Internal terminals
  - External terminals

$$T_{A \rightarrow C} = T_{AB} + T_{BC} - T_B - T_{ABC}$$

- Applying Rent's rule

$$T_{A \rightarrow C} = t \left[ (1 + C_B)^P + (C_B + C_C)^P - C_B^P - (1 + C_B + C_C)^P \right]$$

$$n_{A \rightarrow C} = \alpha T_{A \rightarrow C}$$



# Non-Uniform Probability Distribution

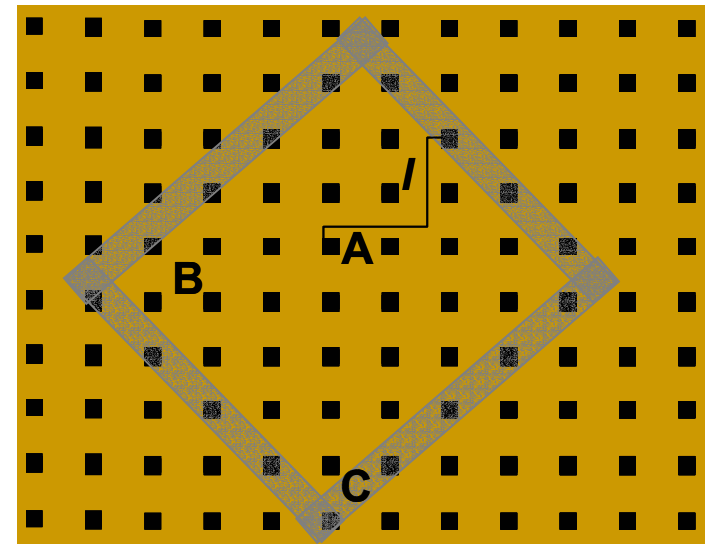
- Number of terminals in  $C_C$ ,  $C_B$ :  $C_B = \sum_{l=1}^{l-1} 4l = 2l(l-1)$ ,  $C_C = 4l$

$$n(l) = \alpha t \left[ (1 + 2l(l-1))^P + (2l(l-1) + 4l)^P - (2l(l-1))^P - (1 + 2l(l-1) + 4l)^P \right]$$

$$q(l) = \frac{1}{2l} \left[ (1 + 2l(l-1))^P + (2l(l-1) + 4l)^P - (2l(l-1))^P - (1 + 2l(l-1) + 4l)^P \right]$$

$$q'(l) \approx P(1-P)2^{-(1-P)} l^{-(4-2P)}$$

- For  $l \gg 1$ :  $q'(l) \approx l^{-(4-2P)}$



# Stroobandt WL Estimation Method

- Modification of Donath method
  - Non-uniform probability distribution
- Interconnection length distribution
  - Structural distribution
  - Occupying probability
- More accurate results than Donath

# Cheng WL Estimation

## Considering IP Block Effect

- Flat WL estimation approach
- Total wirelength
  - Redistribution
  - Detour
- Based on geometrical characteristics
- Provide guidelines for revising floorplanning/global placement before detailed placement and routing

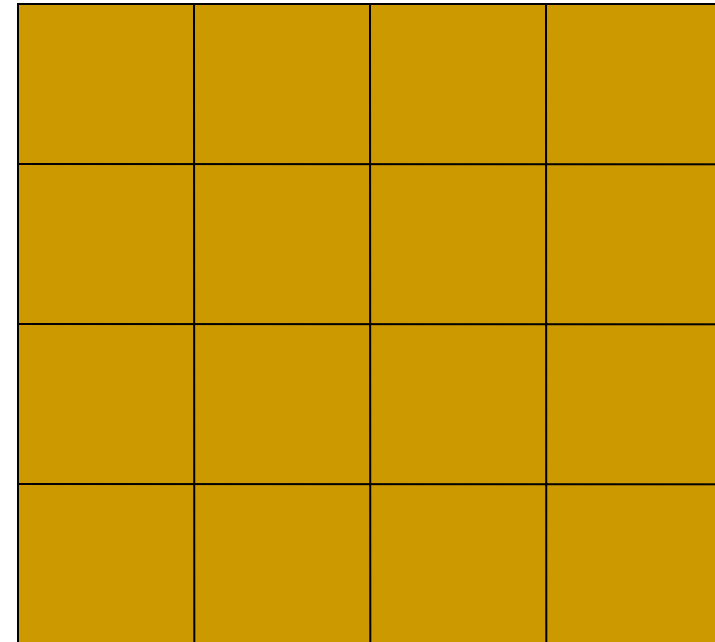
# Taraneh WL Estimation

## Considering IP Block Effect

- Hierarchical approach
  - Consider complexity of circuit using Rent exponent
  - Consider geometrical characteristics
- Deploy effect of different placement alg. in Rent exponent extraction
- Good for large-scale circuits

# Methodology

- Physically partitioning into 4 sub-circuits
- Continue till no. of cells  $\leq \beta$
- At each level of hierarchy
  - $n_h$ : Avg. no. of interconnections
  - $L_h$ : Average length of interconnections

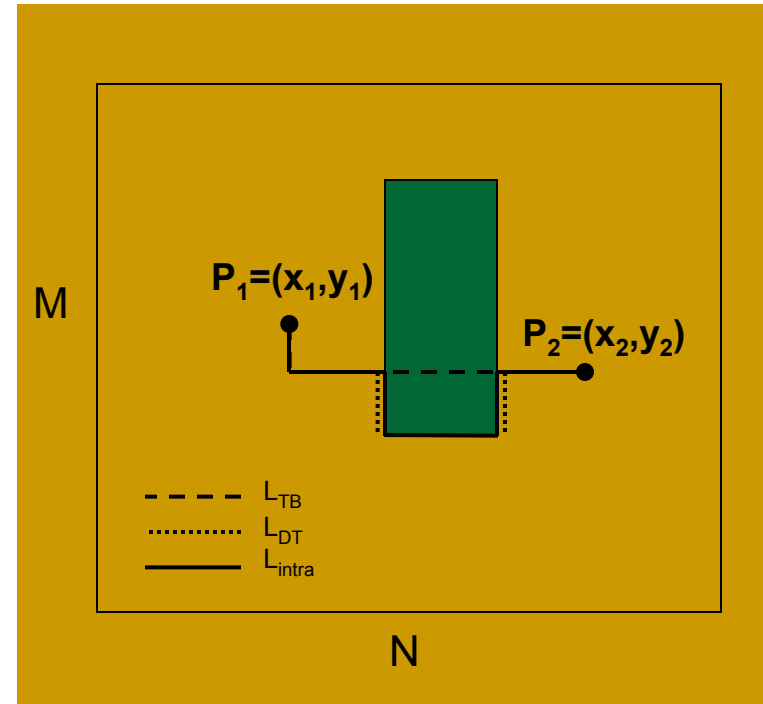


**Total wirelength:** 
$$L_{tot} = \sum_{h=0}^H n_h L_h$$

# Average Length of Interconnection

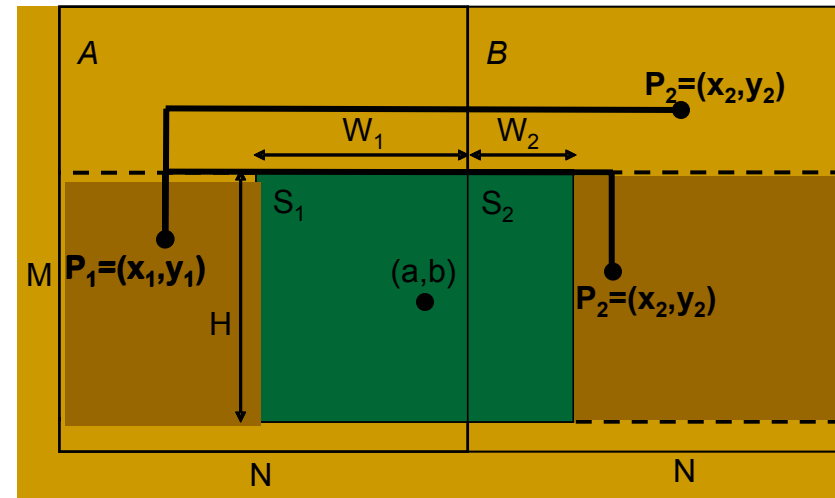
- $L_{TB}$  : WL, transparent blockage
- $L_{DT}^v$  : Vertical detour
- $L_{DT}^h$  : Horizontal detour
- Average WL

$$\bar{L}_{intra} = \bar{L}_{TB} + \bar{L}_{DT}^h + \bar{L}_{DT}^v$$



# Average Length of Interconnection

- Horizontally, vertically, diagonally adjacent bins
  - Transparent block
  - Horizontal detour
  - Vertical detour
- Detour happens if two terminals are on different sides of IP Block
  - Probabilistic nature

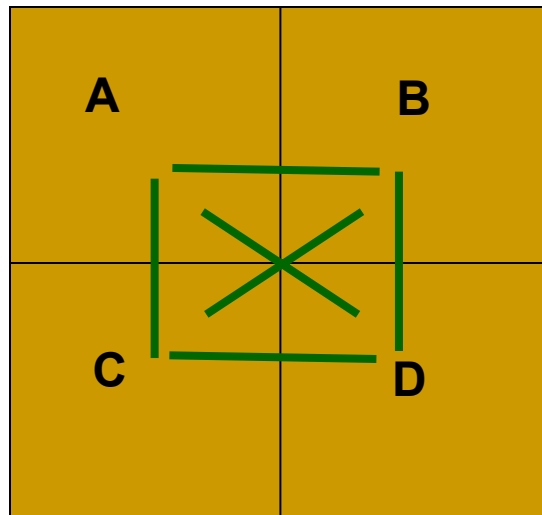




# Average Estimated WL per Level

$$\bar{L}_{\text{inter},l} = \frac{1}{6} \left( \delta \left( L_{\text{inter}}^h(A, B) + L_{\text{inter}}^h(C, D) + L_{\text{inter}}^v(A, C) + L_{\text{inter}}^h(B, D) \right) + (1 - \delta) \left( L_{\text{inter}}^d(A, D) + L_{\text{inter}}^d(B, C) \right) \right)$$

$\delta$  captures optimization behavior of placement algorithms



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# Open Research Problems

- Study the effect of IP blocks on Rent exponent
- Considering the effect of congestion around IP blocks and slivers
- Extending WL estimation for timing driven placement methods

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# Routing Congestion Prediction

- Several post-placement algorithms presented to analyze congestion before routing.
- Use routing estimation model to predict routing congestion
- Derive the mathematical equations to estimate the congestion using a normal distribution approximation
- Use a probabilistic approach to estimate the congestion

# Congestion Prediction

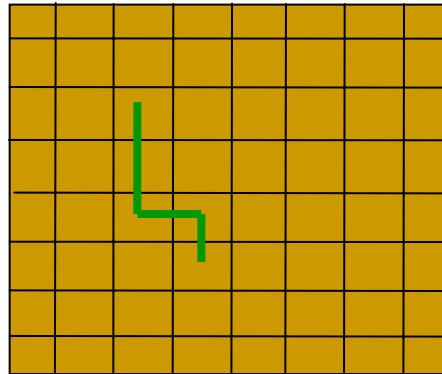
## Concurrent Congestion and Wirelength Estimation

- High-utilization benchmarks
  - Congestion is very likely
  - Detour b/c of congestion
- WL estimation needs congestion map
  - Modeling congested areas as artificial IP blocks
- Regional congestion estimation
  - Needs wirelength estimation for internal routing demand
- Concurrent congestion & wirelength estimation
  - Hierarchical bottom-up approach

# Congestion Prediction

## Regional Congestion Estimation

- Routing demand:  $D(r) = ID(r) + ED(r)$ 
  - Internal routing demand  $ID(r)$
  - External routing demand  $ED(r)$
- Internal routing demand
  - Equal to WL



# Congestion Prediction

## Regional Congestion Estimation

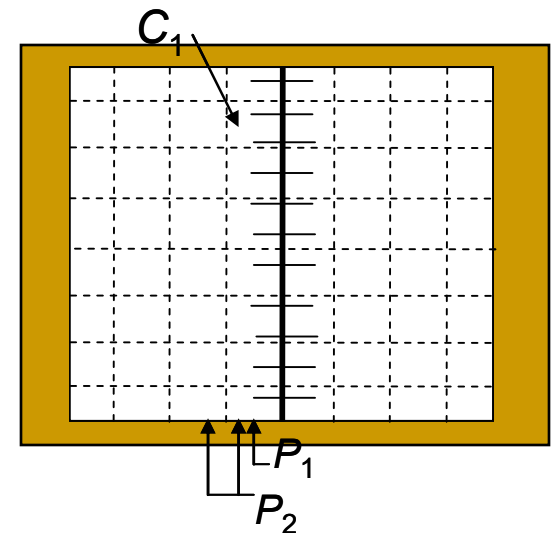
### ■ External Routing Demand

□ The probability of occurring a wire of length  $l$ ,  $P_l = l^{-(4-2P)}$

□ If  $NB = \#$  of bins  $\longrightarrow \sum_{i=2}^{2\sqrt{NB}} P_i + P_1 = 1$

□ And, we have  $\longrightarrow \frac{P_{i+1}}{P_i} = \frac{(i+1)^{-(4-2P)}}{(i)^{-(4-2P)}}$

□ Calculate  $P_1, \dots, P_{2\sqrt{NB}}$



# Congestion Prediction

## Regional Congestion Estimation

### External Routing Demand

- Calculate  $P_1, \dots, P_{2\sqrt{NB}}$

$$\sum_{i=1}^{2\sqrt{NB}} P_i = 1$$

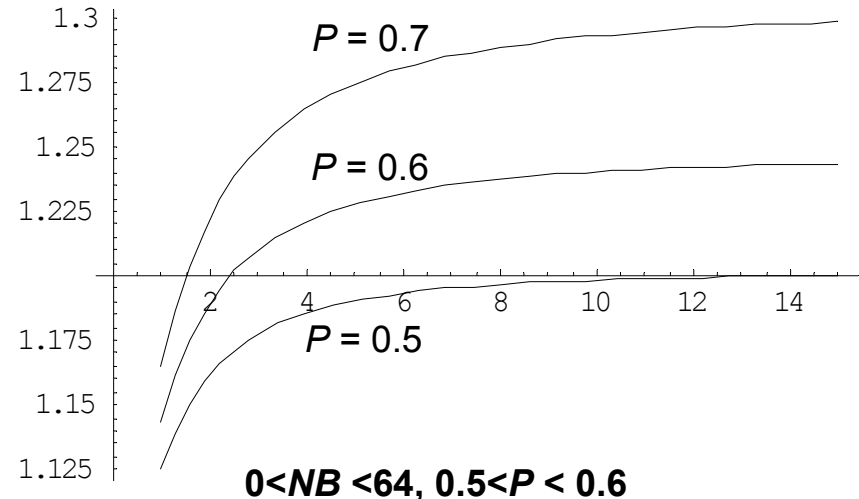
$$P_2 \sum_{i=1}^{2\sqrt{NB}} (i+1)^{-(4-2P)} + P_1 = 1$$

- Riemann Zeta function:

$$\zeta(x) = \frac{1}{\Gamma(x)} \sum_0^{\infty} \frac{u^{x-1}}{e^u - 1} du$$

$$\sum_{i=0}^{2\sqrt{NB}-1} (i+1)^{-(4-2P)} = \text{Zeta}(4-2P) - \text{Zeta}[4-2P, 2\sqrt{NB} + 1]$$

- Total external demand:  $ED = \sum_{i=1}^{2\sqrt{NB}} C_i \times P_i$





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# Concurrent WL and Cong. Prediction

- Bottom-up hierarchical analysis
- Congestion estimation
  - Analyze external routing demand
  - Analyze internal routing demand using WL of previous level
  - Obtain congestion map of this level
- WL Estimation
  - Obtain avg. no. of interconnect per level
  - Analyze avg. length of interconnect
  - Model congested areas (known from previous level) as artificial IP blocks

# Open Research Problems

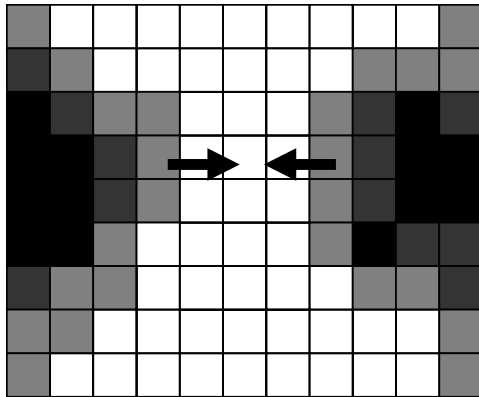
- Considering the effect of vias in the estimation
- Extending congestion estimation for 3D placement
- Extending the concurrent estimation methods for congestion and wirelength on the other estimation methods

# Outline

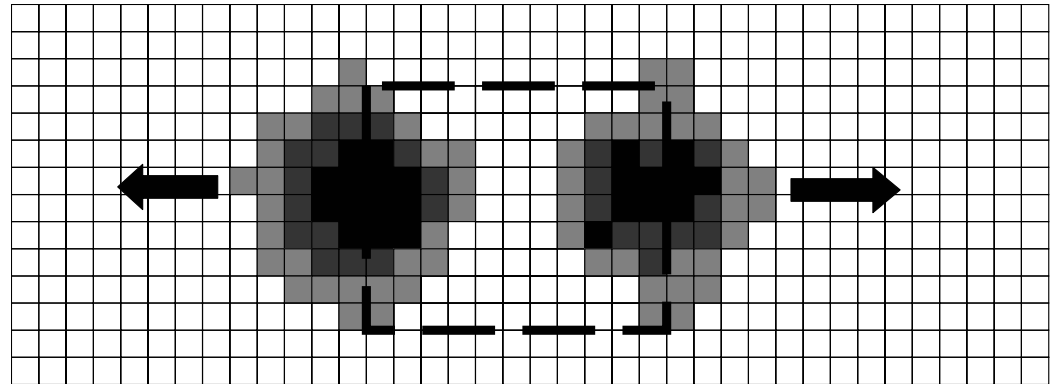
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# Density Utilization Alleviation

- Need a global view of density (congestion) map



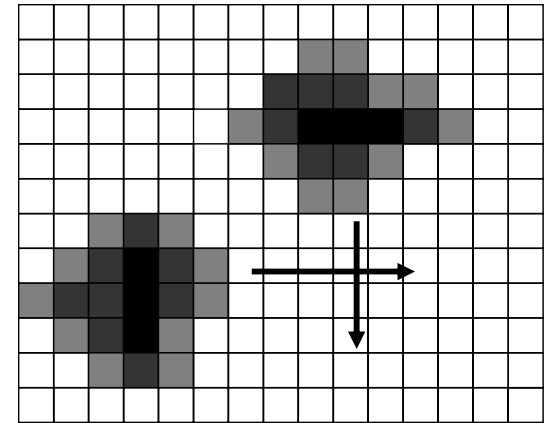
Local density alleviation



Global density alleviation

# Density Utilization Alleviation

- Flow algorithms
  - Local view
  - Ignore WL optimization
- Solution
  - White space allocation
  - Linear/quadratic
  - Greedy



**The interfering flow in the crossing of two arrows**

# Density Utilization Alleviation

## ■ Cell Redistribution

- Form working area round highly-congested bins
- Redistribute cells in working ( by using min-cut partitioning)

## ■ Cell Migration

- Move cells out of highly-congested bins
- Accept move if reduces congestion
- Different Criteria to accept a move

# Open Research Problems

- Incorporating the issues like
  - Thermal placement and hot spot distribution in congestion alleviation algorithms.
- Considering the white space needed around the big IP blocks in the density alleviation algorithms
  - Guarantee routability in later phases.



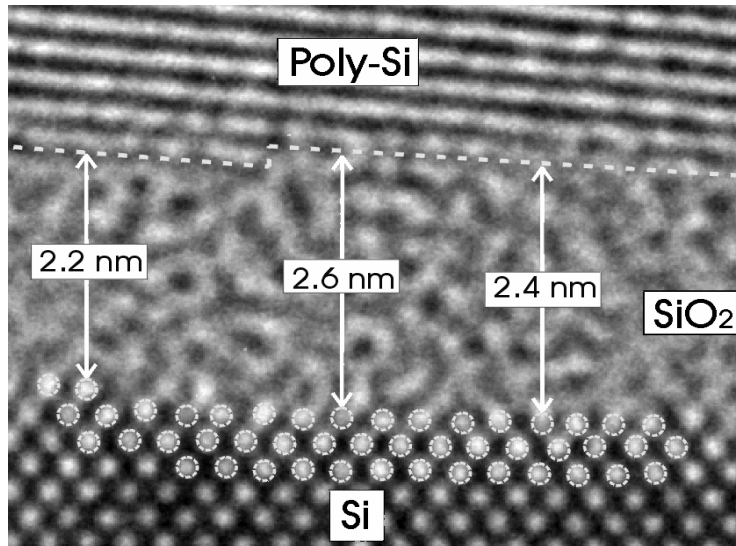
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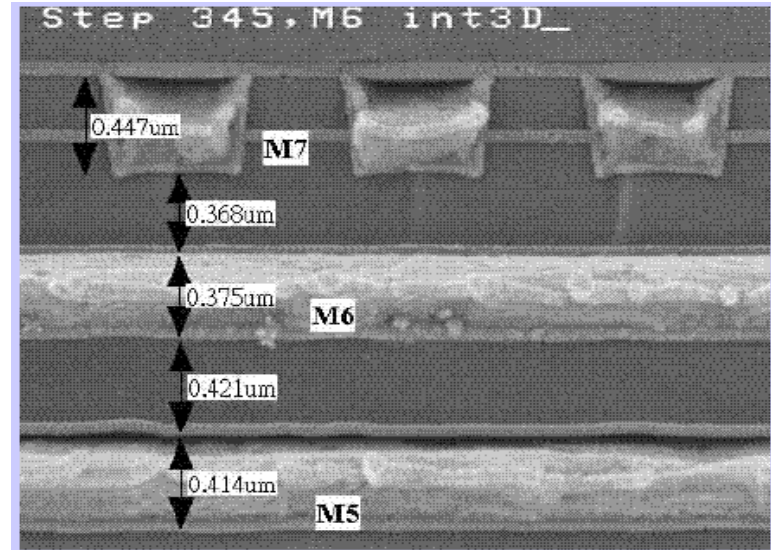
# Statistical Timing Analysis

- Key parameters in device variations
  - Length  $L$ ,
  - Threshold voltage  $V_t$
  - Oxide thickness  $t_{ox}$
- Process parameter variations normally distributed as random variables
- Gate delay,  $d$ , will have a probability distribution function: Gaussian distribution with parameters  $\mu$  and  $\sigma$ .

# Variation Sources



$t_{\text{ox}}$



Wire Thickness

Target=0.375μm

- Figures are courtesy of IBM, Intel and TSMC

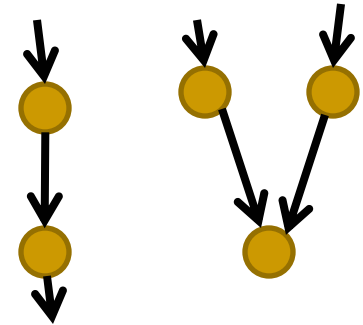
# Gate Delay Distribution

- Gate Delay: *almost* worst case delay

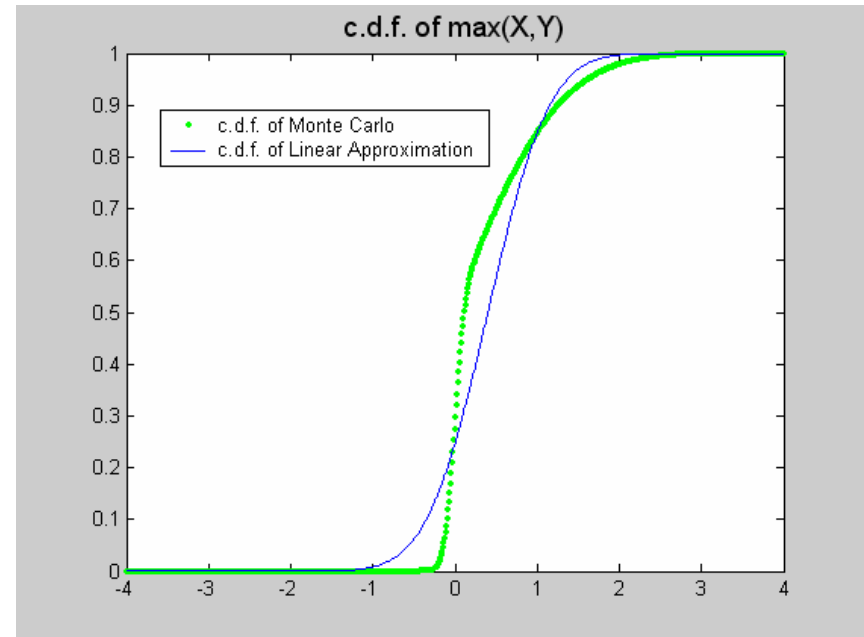
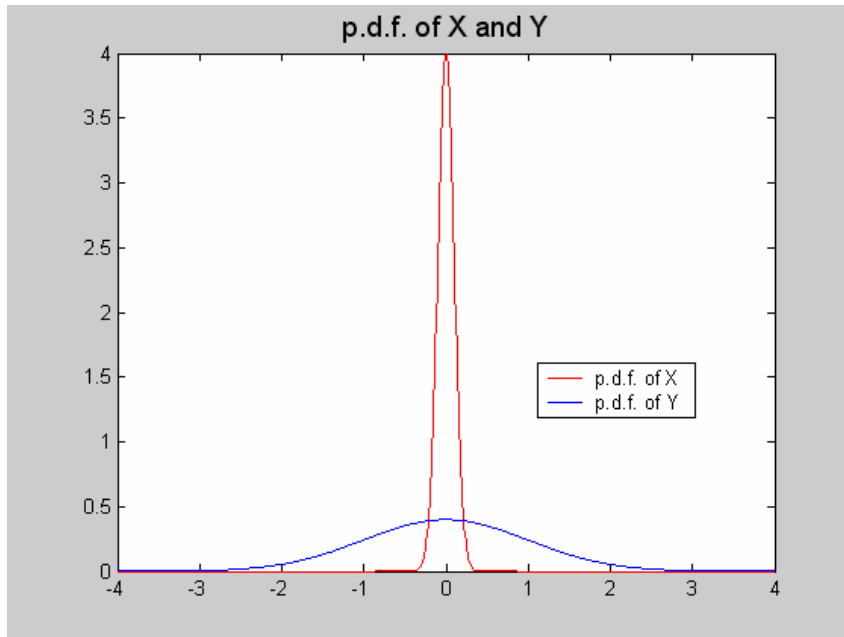
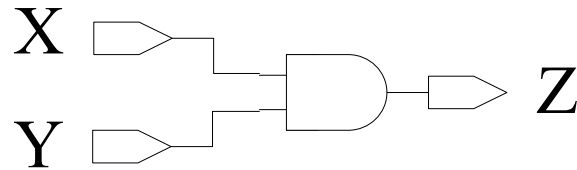
$$\text{prob}(d > D_{gate}) < 0.05$$

- Delay distribution of a circuit using two operators: Max and Sum
- Re-convergent paths produce correlations between delay
- No set of distributions which is closed under both sum and max
- Quantization can be applied to work with discrete distributions with a polynomial associated:

$$G_d(x) = \sum_{i=1} P_i x^{d_i}$$



# MAX operation: inherently non-linear



# Timing Model

- Canonical timing model for gate/wire delay
  - Assumption: Gate/wire delay is linearly dependent on the variation sources
  - Problem: Non-linear dependency of delay on process parameters
- Linear MAX approximation for circuit delay
  - Problem: Significant error could occur in some cases

# Open Problems

- Obtain a probabilistic model nets with re-convergent fan-outs
  - Half perimeter approximation no longer accurate because of the branches in a net
- Analyze delay distribution for nets during placement.
- Incorporating delay distribution into placement algorithm in timing driven placement

# Outline

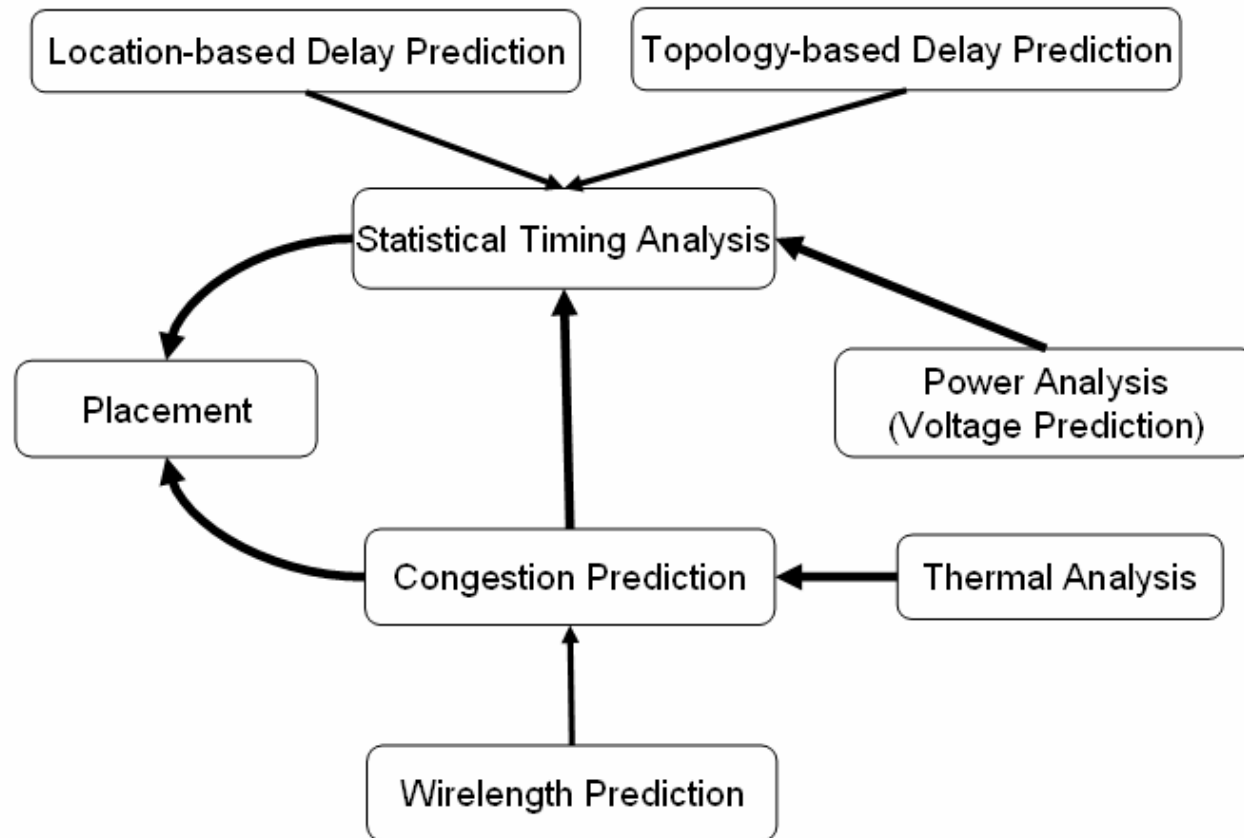
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# Integrated Cong. & Delay Estimation

- Hierarchical CAD flows to consider wirelength, congestion, thermal issues and power early in design flow
  - Pure wirelength minimization without congestion estimation methods may lead to un-routable designs
  - Wirelength minimization along with congestion removal but ignoring thermal issues may lead to designs with several hot-spots.
- All these predictions should be highly integrated
  - Goal is to degrade the harmful side effects of optimizing each parameter individually

# Integrated Cong. & Delay Estimation



# Conclusion

- Integrated prediction methods
- WL estimation
  - Individual net length
  - Total interconnect length
- Congestion Estimation
  - Pre-Placement
  - Post-Placement
- Timing Analysis
- Integrated Estimation of WL, Congestion, and Delay

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# Thank You!